4.6

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A: amount in the account (dollars)
P: principle
r: interest rate (decimal)
n: number of times per year interest is compounded
t: time of investment (years)

Example:
If $22,000 is invested at an annual rate of 4.5% and compounded 2 times per year, find the balance after 2 years and 10 years.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $\quad 4.5\% = 0.045$

a) 2 years

$P = 22000$
$r = 0.045$
$n = 2$
$t = 2$

$A = 22000\left(1 + \frac{0.045}{2}\right)^{(2*2)}$

$A = \$24047.83$

b) 10 years

$P = 22000$
$r = 0.045$
$n = 2$
$t = 10$

$A = 22000\left(1 + \frac{0.045}{2}\right)^{(2*10)}$

$A = \$34,331.20$

Continuous Compounding Interest Formula:

$$A = Pe^{rt}$$

A: amount in account (dollars)
P: principle
r: interest rate (as a decimal)
t: time of investment (years)

Example:
If $22,000 is invested at an annual rate of 4.5% and compounded continuously, find the balance after 2 years and 10 years.

$A = Pe^{rt}$

a) 2 years

$P = 22000$
$r = 0.045$
$t = 2$

$A = 22000e^{(0.045*2)}$
$A = \$24071.83$

b) 10 years

$t = 10$

$A = 22000e^{(0.045*10)}$
$A = \$34502.87$

Find the time required for money invested at an annual rate of 8% to triple in value if the investment is:

a. compounded monthly

b. compounded continuously

a) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$r = 0.08$

$n = 12$

$P = 1$

$A = 3$

$$\frac{3}{1} = \frac{1\left(1+\frac{0.08}{12}\right)^{12t}}{1}$$

$$3 = \left(1+\frac{0.08}{12}\right)^{12t}$$

$$\log 3 = \log\left(1+\frac{0.08}{12}\right)^{12t}$$

$$\frac{\log 3}{12\log\left(1+\frac{0.08}{12}\right)} = \frac{12t\log\left(1+\frac{0.08}{12}\right)}{12\log\left(1+\frac{0.08}{12}\right)}$$

$$\frac{\log 3}{\left(12\log\left(1+\frac{0.08}{12}\right)\right)} = t$$

13.778 years = t

b) $A = Pe^{rt}$

$r = 0.08$

$P = 1$

$A = 3$

$$\frac{3}{1} = \frac{1e^{0.08t}}{1}$$

$$3 = e^{0.08t}$$

$$\log 3 = \log\left(e^{0.08t}\right)$$

$$\frac{\log 3}{0.08\log(e)} = \frac{0.08t\log(e)}{0.08\log(e)}$$

$$\frac{\log 3}{0.08\log(e)} = t$$

13.733 years = t

Assuming that air resistance is porportional to velocity, the velocity, v, in feet per second, of a falling object after t seconds is given by
v = 64(1 - e^(-t/2)).

1. What is the velocity of the falling object after 15 seconds?

$y_1 = 64(1 - e^{-x/2})$

$x = 15$

x: seconds
y: velocity

$y_1(15) =$ 63.965 ft/sec

2. Determine when the velocity is 50 feet per second.

$y_2 = 50$

$x = 3.040$

Window: x: 0, 15
Zoom 0

3.040 sec