Name ________________________________ MAT 120 – Chapter 8

Hypothesis Testing

Statistical Hypothesis: conjecture about a population parameter.

Null Hypothesis ($H_0$): a statistical hypothesis that states that there is **no difference** between a parameter and a specific value, or that there is **no difference** between two parameters.

Alternative Hypothesis ($H_A$): statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there **is a difference** between two parameters.

Example 1
State the null and alternative hypotheses for the following conjectures:

1. A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.4 pounds.

2. An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1,000 is 16.

3. A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 84.

4. The average age of community college students is 23.1 years.

Statistical Test: uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

Test Value: the numerical value obtained from a statistical test.

Type I Error: rejecting the null hypothesis when it is true.

Type II Error: failing to reject the null hypothesis when it is false.

Example 2
What are the type I and type II errors in the following situations:

a. In a jury trial, there are four possible outcomes. The defendant is either guilty or innocent, and he or she will be convicted or acquitted.
   $H_0$: The defendant is innocent
   $H_A$: The defendant is not innocent (guilty)

b. $H_0$: Smoking cigarettes causes cancer
   $H_A$: Smoking cigarettes does not cause cancer

Level of Significance: maximum probability of committing a type I error. This probability is symbolized by α. The P(type I error) = α.

Homework: pg.405 #13

Z Test for a Mean

- used to test the mean of a **large sample** ($n \geq 30$)

**Traditional Method**

The ***z* test** is a statistical test for the mean of a population. It can be used when $n \geq 30$ or when the population is normally distributed and $\sigma$ is known.

The formula for the *z* test is

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

where

$\overline{X}$ = sample mean
$\mu$ = hypothesized population mean
$\sigma$ = population standard deviation
$n$ = sample size

**Five Steps for Solving Hypothesis-Testing Problems:**

1. State the hypotheses and identify the claim
2. Find the critical value(s)
3. Compute the test value
4. Make the decision to reject or not reject the null hypothesis
5. Summarize the results

Example 3

A researcher reports that the average salary of assistant professors is more than $43,000. A sample of 30 assistant professors has a mean salary of $44,230. At $\alpha = 0.10$, test the claim that the assistant professors earn more than $43,000 a year. The standard deviation of the population is $5,080.

Example 4

A medical foundation reports that the average cost of rehabilitation for stroke victims is \$24,305. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selected a random sample of 34 stroke victims at the hospital and found that the average cost of their rehabilitation is \$25,115. The standard deviation of the population is \$3,230. At $\alpha = 0.05$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from \$24,305?

Example 5

A survey claims that the average cost of a hotel room in a city is less than $69.36. To test the claim, a researcher selects a sample of 30 hotel rooms and finds that the average cost is $68.44. The standard deviation of the population is $3.56. At $\alpha = 0.05$, is there enough evidence to reject the claim?

Example 6

A researcher wishes to test the claim that the average age of lifeguards in a coastal town is greater than 21 years. She selects a sample of 35 guards and finds the mean of the sample to be 21.7 years, with a standard deviation of 2 years. Is there evidence to support the claim at $\alpha = 0.05$?

**P-Value Method**

**Five Steps for Solving Hypothesis-Testing Problems (P-Value Method):**

1. State the hypotheses and identify the claim
2. Compute the test value
3. Find the p-value
4. Make the decision
5. Summarize the results

**Decision Rule When Using a *P*-Value**

If $P$-value $\leq \alpha$, reject the null hypothesis.

If $P$-value $> \alpha$, do not reject the null hypothesis.

Example 7

A researcher wishes to test the claim that the average age of lifeguards in a coastal town is greater than 21 years. She selects a sample of 35 guards and finds the mean of the sample to be 21.7 years, with a standard deviation of 2 years. Find the *P*−value. Is there evidence to support the claim at $\alpha = 0.05$?

Example 8

A researcher claims that the average wind speed in a certain city is 5 miles per hour. A sample of 33 days has an average wind speed of 5.3 miles per hour. The standard deviation of the sample is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*−value method.

Example 9

A research report states that full-time Ph.D. students in a certain state receive an average salary of $16,200. The dean of graduate studies at a large university in the state feels that Ph.D students at his school earn more than this. He surveys 37 randomly selected students and finds that their average salary is $16,390, with a standard deviation of $720. At $\alpha = 0.07$, is there enough evidence to reject the claim?

Homework: pg. 414 #1, 2, 5, 9, 15, 17, 19, 21, 23 t Test for a Mean

- used to test the mean of a **small sample** (n<30)

The ***t* test** is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, $\sigma$ is unknown, and $n < 30$.

The formula for the *t* test is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. $= n - 1$.

Example 1

A job placement director claims that the average starting salary for nurses is $21,000. A sample of 10 nurses has a mean of $20,600 and a standard deviation of $520. Is there enough evidence to reject the director's claim at $\alpha = 0.05$?

Example 2

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 38.6 milliliters per kilogram (ml/kg) and a standard deviation of 7 ml/kg. If the average of all adults is 35 ml/kg, is there enough evidence to support the physicians' claim at $\alpha = 0.05$?

Homework: pg. 426 #5, 7, 11, 13