Name ________________________________ MAT 120 – Chapter 7

Confidence Intervals for the Mean (μ known or **n ≥ 30**) and Sample Size

Point Estimate: specific numerical value estimate of a parameter. The best point estimate of the population mean μ is the sample mean $\overline{X}$.

Interval Estimate: an interval or a range of values used to estimate a parameter. This estimate may or may not contain the value of the parameter being estimated.

Confidence Level: probability that the interval estimate will contain the parameter, assuming that a large number of samples are selected and that the estimation process of the same parameter is repeated.

Confidence Interval: specific interval estimate of a parameter determined by using data obtained from a sample and by using the specific confidence level of the estimate.

Maximum Error of Estimate: the maximum likely difference between the point estimate of a parameter and the actual value of the parameter.

Formula for the Confidence Interval of the Mean for a Specific α

$$\overline{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \overline{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

For a 90% confidence interval, $z_{\alpha/2} = 1.65$; for a 95% confidence interval, $z_{\alpha/2} = 1.96$; and for a 99% confidence interval, $z_{\alpha/2} = 2.58$.

The values for the z distribution are found in Table E of Appendix C.

Example 1
A researcher wishes to estimate the average amount of money a person spends on lottery tickets each month. A sample of 90 people who play the lottery found the mean to be $23.60 and the standard deviation to be 3. Find the 95% confidence interval of the population mean.

Example 2

A survey of 70 adults found that the mean age of a person's primary vehicle is 5.6 years. Assuming the standard deviation of the population is 0.5 year; find the 99% confidence interval of the population mean.

Example 3

A sample of the reading scores of 31 fifth-graders has a mean of 79. The standard deviation of the sample is 13. Find the best point estimate of the mean. Find the 95% confidence interval of the mean reading scores of all fifth-graders.

Example 4

The following data represent a sample of the assets (in millions of dollars) of 30 banks in a state. Find the 99% confidence interval of the mean.

12.48	**16.4**	**4.47**
2.8	**1.21**	**2.17**
13.19	**9.27**	**1.39**
73.14	**1.89**	**14.54**
11.57	**6.63**	**1.02**
8.6	**3.27**	**18.16**
7.55	**4.76**	**16.54**
40.38	**2.55**	**21.68**
5.06	**1.49**	**12.12**
2.35	**12.79**	**2.8**

Example 5

Find the 90% confidence interval of the population mean for the incomes of a group of credit unions. A random sample of 30 credit unions is shown. The data are in thousands of dollars.

4,240	**2,962**	**1,677**	**444**	**91**	**2,214**
4,768	**3,387**	**2,207**	**3,976**	**2,219**	**1,224**
1,669	**3,409**	**2,481**	**4,326**	**160**	**1,961**
3,727	**1,047**	**4,749**	**2,972**	**4,618**	**4,365**
2,040	**1,625**	**3,568**	**3,699**	**3,617**	**688**

Formula for the Minimum Sample Size Needed for an Interval Estimate of the Population Mean

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

where E is the maximum error of estimate. If necessary, round the answer up to obtain a whole number. That is, if there is any fraction or decimal portion in the answer, use the next whole number for sample size n.

Example 6

The college president asks the statistics teacher to estimate the average age of the students at their college. How large a sample is necessary? The statistics teacher would like to be 99% confident that the estimate should be accurate within 0.8 year. From the previous study, the standard deviation of the ages is known to be 2.7 years.

Example 7

A university dean of students wishes to estimate the average number of hours students spend doing homework per week. The standard deviation from a previous study is 2.6 hours. How large a sample must be selected if he wants to be 99% confident of finding whether the true mean differs from the sample mean by 1.5 hours?

Homework: pg. 358 #11, 13, 17, 23, 25

<u>Confidence Intervals for the Mean **(n < 30)** when μ is unknown</u>

- Need to use the t-distribution

<u>Characteristics of the t Distribution</u>

The t distribution shares some characteristics of the normal distribution and differs from it in others. The t distribution is similar to the standard normal distribution in these ways.

1. It is bell-shaped.
2. It is symmetric about the mean.
3. The mean, median, and mode are equal to 0 and are located at the center of the distribution.
4. The curve never touches the x axis.

The t distribution differs from the standard normal distribution in the following ways.

1. The variance is greater than 1.
2. The t distribution is actually a family of curves based on the concept of degrees of freedom, which is related to sample size.
3. As the sample size increases, the t distribution approaches the standard normal distribution.

Formula for a Specific Confidence Interval for the Mean When σ Is Unknown and $n < 30$

$$\bar{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

The degrees of freedom are $n - 1$.

The values for the t distribution are found in Table F in Appendix C.

Example 1

Fifteen randomly selected automobiles were stopped, and the tread depth of the right front tire was measured. The mean was 0.35 inch, and the standard deviation was 0.08 inch. Find the 95% confidence interval of the mean depth. Assume the variable is approximately normally distributed.

Example 2
The average hemoglobin reading for a sample of 23 teachers was 11 grams per 100 milliliters, with a sample standard deviation of 3 grams. Find the 99% confidence interval of the true mean. Assume that the variable is approximately normally distributed.

Example 3
The data represents a sample of the number of home fires started by candles for several years. Find the 99% confidence interval for the mean number of home fires started by candles each year.

5,440 5,904 6,089 6,311 7,157 8,439 9,928

Example 4
A random sample of the number of barrels (in millions) of oil produced per day by world countries is listed below. Estimate the mean oil production with 95% confidence. Round intermediate steps and final answer to one decimal place.

5,455 5,911 6,082 6,312 7,145 8,441 9,931

Homework: pg.366 #5, 7, 9, 13, 17, 19

Confidence Intervals and Sample Size for Proportions

Symbols Used in Proportion Notation

p = population proportion

$\hat{p}$ (read "p hat") = sample proportion

For a sample proportion,

$$\hat{p} = \frac{X}{n} \quad \text{and} \quad \hat{q} = \frac{n - X}{n} \quad \text{or} \quad 1 - \hat{p}$$

where X = number of sample units that possess the characteristics of interest and n = sample size.

Example 1
In a recent survey of 100 households, 51 had central air conditioning. Find $\hat{p}$ and $\hat{q}$, where $\hat{p}$ is the proportion of households that have central air conditioning. What is $\hat{q}$?

Formula for a Specific Confidence Interval for a Proportion

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

when np and nq are each greater than or equal to 5.

Example 2
A sample of 500 nursing applications included 65 from men. Find the 90% confidence interval of the true proportion of men who applied to the nursing program.

Example 3

A survey of 130,000 boat owners found that 10% of the pleasure boats were named *Odyssey*. Find the 95% confidence interval of the true proportion of boats named *Odyssey*.

Example 4

In a certain county school district, a survey of 110 students showed that 20% carried their lunches to school. Find the 95% confidence interval of the true proportion of students who carried their lunches to school.

Formula for Minimum Sample Size Needed for Interval Estimate of a Population Proportion

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

If necessary, round up to obtain a whole number.

Example 5
A researcher wishes to estimate, with 95% confidence, the proportion of people who own a home computer. A previous study shows that 46% of those interviewed had a computer at home. The researcher wishes to be accurate within 3% of the true proportion. Find the minimum sample size necessary.

Example 6
A medical researcher wishes to determine the percentage of females who take vitamins. He wishes to be 99% confident that the estimate is within 2% of the true proportion. A recent study of 238 females showed that 38% took vitamins. How large should the sample size be?

Homework: pg.374 #1, 3, 5, 7, 11, 13, 15, 17, 19