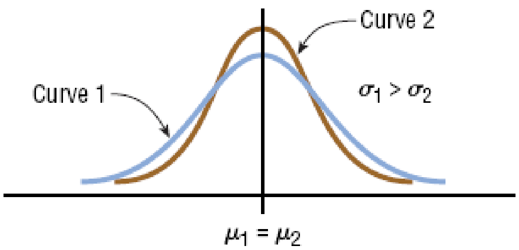
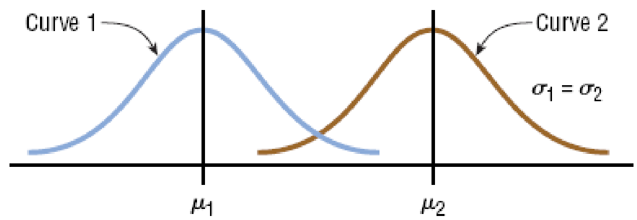
Name ________________________________ MAT 120 – Chapter 6

Normal Distribution

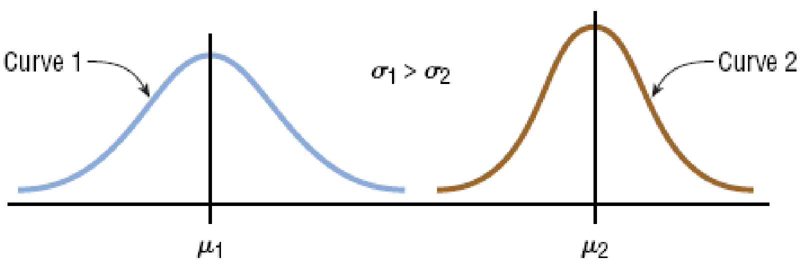
1. The normal distribution is a continuous, symmetric, bell-shaped distribution of a variable.
2. The shape and position of a normal distribution curve depend on two parameters: the mean and the standard deviation.



(a) Same means but different standard deviations



(b) Different means but same standard deviations



(c) Different means and different standard deviations

3. The area under the curve that lies within:
   a. 1 standard deviation is **68%**
   b. 2 standard deviations is **95%**
   c. 3 standard deviations is **99.7%**

Standard Normal Distribution: a normal distribution with a mean of 0 and a standard deviation of 1.

*All normally distributed variables can be transformed into the standard normally distributed variable by using the formula for the standard score:

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

Example 1
Find the area under the normal distribution curve between $z = 0$ and $z = 2.32$.

Example 2
Find the area under the normal distribution curve between $z = 0$ and $z = 1.5$.

Example 3
Find the area under the normal distribution curve between $z = 0$ and $z = -1.73$.

Example 4
Find the area under the normal distribution curve to the right of $z = 1.29$.

Example 5
Find the area under the normal distribution curve to the left of $z = -1.91$.

Example 6
Find the area under the normal distribution curve between $z_1 = 1.00$ and $z_2 = 2.54$.

Example 7
Find the area under the standard normal distribution curve between $z = 1.83$ and $z = 1.94$.

Example 8
Find the area under the normal distribution curve between $z_1 = -2.41$ and $z_2 = -0.88$.

Example 9
Find the area under the normal distribution curve between $z_1 = +1.6$ and $z_2 = -1.34$.

Example 10
Find the area under the normal distribution curve to the left of $z = 2.02$.

Example 11
Find the area under the normal distribution curve to the right of $z_1 = +1.33$ and to the left of $z_2 = -2.98$.

Example 12
Find the following probability using the standard normal distribution.
$P(0 < z < 2.31)$

Example 13
Find the following probability using the standard normal distribution.
$P(z < 1.63)$

Example 14
Find the following probability using the standard normal distribution.
$P(z > 1.17)$

Example 15
Find the following probability using the standard normal distribution.
$P(-2.35 < z < 0)$

Example 16
Find the following probability using the standard normal distribution.
$P(z < 1.25)$

Example 17
Find the z value such that the area under the standard normal distribution curve between 0 and the z value is 0.2088.

Homework: pg. 302 #3, 4, 5, 7 – 39 (every other odd), 41, 43, 45

Applications of the Normal Distribution

Example 1

Suppose the mean number of hours a worker spends on the computer is 3.1 hours per workday. Assume the standard deviation is 0.5 hour. Find the percentage of workers who spend less than 3.6 hours on the computer. Assume the variable is normally distributed.

Example 2

Suppose each month, a household generates an average of 28 pounds of newspaper for garbage or recycling. Assume the standard deviation is 2 pounds. If a household is selected at random, find the probability of its generating between 27 and 32 pounds per month. Assume the variable is approximately normally distributed.

Example 3

Suppose an Automobile Association reports that the average time it takes to respond to an emergency call is 25 minutes. Assume the variable is approximately normally distributed and the standard deviation is 4.5 minutes. If 90 calls are randomly selected, approximately how many will be responded to in less than 19 minutes?

Example 4
The average salary for first-year teachers is $26,891. If the distribution is approximately normal with a standard deviation of $2,600, what is the probability that a randomly selected first-year teacher makes less than $21,249 a year?

Example 5
The average charitable contribution itemized per income tax return in Pennsylvania is $731. Suppose that the distribution of contributions is normal with standard deviation of $99. Find the limits for the middle 70% of contributions.

Example 6
Suppose the average age of a passenger train car is 19.4 years. If the distribution of ages is normal and 10% of the cars are older than 22.9 years, find the standard deviation.

Example 7
If a one-person household spends an average $785 per week on groceries, find the minimum dollar amount spent per week for the middle 50% of one-person households. Assume that the standard deviation is $108 and the variable is normally distributed.

Example 8
To qualify for a police academy, candidates must score in the top 5% on a general abilities test. The test has a mean of 200 and a standard deviation of 45. Find the lowest possible score to qualify. Assume the test scores are normally distributed.

Example 9
For a medical study, a researcher wishes to select people in the middle 64% of the population based on blood pressure. If the mean systolic blood pressure is 122 and the standard deviation is 6, find the upper and lower readings that would qualify people to participate in the study.

Homework: pg. 316 #1, 3, 5, 7, 13, 15, 17, 21, 25, 29, 31, 53

The Central Limit Theorem

<u>Sampling Distribution of Sample Means</u>: distribution using the means computed from all possible random samples of a specific size taken from a population

<u>Central Limit Theorem</u>: As the sample size *n* increases without limit, the shape of the distribution of the sample means taken with replacement from a population with mean μ and standard deviation μ\σ will approach a normal distribution. This distribution will have a mean μ and a standard deviation σ/ $\sqrt{n}$. To calculate the z-score when dealing with sample means: $z = \dfrac{\overline{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

Example 1
A research company reported that children between the ages of 2 and 7 watch an average of 26 hours of television per week. Assume the variable is normally distributed and the standard deviation is 2 hours. If 20 children between the ages of 2 and 7 are randomly selected, find the probability that the mean of the number of hours they watch television will be greater than 27.3 hours.

Example 2
Suppose the average age of a registered vehicle is 9 years, or 108 months. Assume the standard deviation is 16 months. If a random sample of 49 vehicles is selected, find the probability that the mean of their age is between 100 and 110 months.

Example 3 – Read Carefully
The average number of pounds of meat that a person consumes a year is 218.5 pounds. Assume that the standard deviation is 27 pounds and the distribution is approximately normal. Find the probability, rounded to four decimal places, that a person selected at random consumes less than 221 pounds per year.

Example 4
The average number of pounds of meat that a person consumes a year is 213.9 pounds. Assume that the standard deviation is 29 pounds and the distribution is approximately normal. If a sample of 40 individuals is selected, find the probability that the mean of the sample will be less than 219 pounds per year.

Example 5
The average yearly cost per household of owning a dog is $219.33. Suppose that we randomly select 40 households that own a dog. What is the probability that the sample mean for these 40 households is greater than $224.00? Assume the standard deviation is $27.

Homework: pg. 329 #9, 11, 15, 17, 19, 21