Name ______________________________ MAT 120 – Chapter 4

Probability and Counting Rules

Probability: Chance of an event occurring
Sample Space: the set of all possible outcomes of a probability experiment
Example 1
Find the sample spaces for:

a. flipping one coin

b. rolling one die

c. flipping two coins

Example 2
Using a tree diagram, find the sample space for:

a. flipping four coins

b. flipping one coin, rolling one die, and flipping another coin

<u>Classical Probability</u>: uses sample spaces to determine the numerical probability that an even will happen.

> The probability of any event E is
>
> $$\frac{\text{Number of outcomes in } E}{\text{Total number of outcomes in the sample space}}$$
>
> This probability is denoted by
>
> $$P(E) = \frac{n(E)}{n(S)}$$
>
> This probability is called *classical probability*, and it uses the sample space S.

Example 3
Find the probability of:

a. tossing a coin and getting a head

b. rolling a die and getting a five

c. picking from a regular deck of cards and getting a king

d. picking from a regular deck of cards and getting a heart

e. picking from a regular deck and getting a three or a diamond

f. picking from a regular deck and getting a three and a diamond

g. picking from a regular deck and getting a face card (jack, queen, or king)

h. a family of three children having all boys

Four basic probability rules:

1. The probability of any even E is a number between and including 0 and 1. This is denoted by $0 \leq P(E) \leq 1$.

2. If an event E cannot occur, its probability is 0.

3. If an event E is certain, then the probability of E is 1.

4. The sum of the probabilities of all the outcomes in the sample space is 1.

<u>Complement of Event *E*</u>: set of outcomes in the sample space that is not included in the outcomes of event *E*. The complement of E is denoted by $\overline{E}$.

Example 4

Find the complement of each event:

a. rolling a die and getting a 2

b. selecting a day of the week and getting a weekend day

Rule for Complementary Events:

$P(\overline{E}) = 1 - P(E)$ or $P(E) = 1 - P(\overline{E})$ or $P(E) + P(\overline{E}) = 1$

Example 5
If the probability that it will rain tomorrow is 0.32, what is the probability that it won't rain tomorrow?

Empirical Probability: relies on actual experience to determine the likelihood of outcomes.

Example 1
In a college class of 250 graduating seniors, 30 have jobs waiting, 10 are going to medical school, 10 are going to law school, and 10 are going to various other kinds of graduate schools. Select one graduate at random. What is the probability that the student is going to graduate school?

Example 2
In a college class of 300 graduating seniors, 60 have jobs waiting, 20 are going to medical school, 10 are going to law school, and 30 are going to various other kinds of graduate schools. Select one graduate at random. What is the probability that the student is going to medical school?

Example 3
For a recent year, 60% of the families in the United States had no children under the age of 18; 16% had one child; 16% had two children; 5% had three children; and 3% had four or more children. If a family is selected at random, find the probability that the family has two or three children.

Example 4
For a specific year a total of 2,823 postal workers were bitten by dogs. The **top six cities** for crunching canines were as follows.

Houston	**36**	**Chicago**	**46**
Miami	**41**	**Los Angeles**	**34**
Brooklyn	**31**	**Cleveland**	**31**

If one bitten postal worker is selected at random, what is the probability that he was bitten in a city other than Houston, Chicago, or Los Angeles?

Example 5
A roulette wheel has 38 spaces numbered 1 through 36, 0, and 00. Find the probability of getting an odd number greater than 28.

Example 6
A researcher asked 23 people if they liked the taste of a new soft drink. The responses were classified as "yes," "no," or "undecided." The results were categorized in a frequency distribution, as shown.

Response	**Frequency**
Yes	**17**
No	**2**
Undecided	**4**
Total	**23**

Find the probability that a person responded no.

Example 7
In a sample of 80 people, 33 had type O blood, 34 had type A blood, 6 had type B blood, and 7 had type AB blood. Find the probability that a person has type B or type AB blood.

Example 8

In a sample of 100 people, 40 had type O blood, 41 had type A blood, 6 had type B blood, and 13 had type AB blood. Find the probability a person does not have type O blood.

Example 9

Hospital records indicated that maternity patients stayed in the hospital for the number of days shown in the distribution.

Number of days stayed	**Frequency**
3	**18**
4	**30**
5	**49**
6	**20**
7	**8**
	125

a. Find the probability that a patient stayed exactly 6 days.

b. Find the probability that a patient stayed less than 6 days.

c. Find the probability that a patient stayed at most 6 days.

Homework: pg. 185 #6, 7, 9, 12, 13, 14, 17, 18, 19, 21, 24, 28, 29, 31, 33, 34

The Addition Rules for Probability

Mutually Exclusive Events: two events cannot happen at the same time (have no events in common)

Example 1
Determine if the following events are mutually exclusive:

a. Drawing a single card from a deck and getting a two and a heart

b. Drawing a single card from a deck and getting a two and a face card

c. Rolling a singe die and getting an odd number and an even number

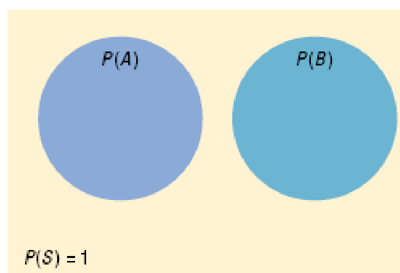
d. Rolling a single die and getting a 2 and an even number

e. Rolling a single die and getting an even number and a number greater than 4

f. Rolling a single die and getting a number greater than three and a number less than three

Addition Rule 1: When two events A and B are mutually exclusive, the probability that A or B will occur is

P(A or B) = P(A) + P(B)



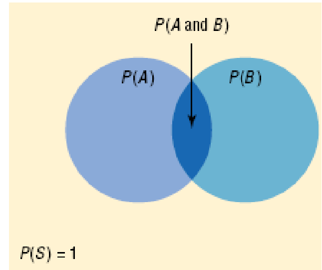
(a) Mutually exclusive events
P(A or B) = P(A) + P(B)

Example 2
A box contains 7 glazed doughnuts, 4 jelly doughnuts, and 9 chocolate doughnuts. If a person selects one doughnut at random, find the probability that it is either a glazed doughnut or a chocolate doughnut.

Example 3
At a political rally, there are 16 Republicans, 17 Democrats, and 4 Independents. If a person is selected at random, find the probability that he or she is either a Democrat or an Independent.

<u>Addition Rule 2</u>: If A and B are NOT mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



b) Non-mutually exclusive events
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 4
A single card is drawn from a deck. Find the probability that it is a five or a spade.

Example 5
In a hospital unit there are 11 nurses and 3 physicians; 9 nurses and 1 physician are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

Example 6
On New Year's Eve, the probability of a person's driving while intoxicated is 0.34, the probability of a person's having a driving accident is 0.14, and the probability of a person's having a driving accident while intoxicated is 0.05. What is the probability of a person's driving while intoxicated or having a driving accident?

Example 7
The probability that a student owns a car is 0.64, and the probability that a student owns a computer is 0.76. If the probability that a student owns both is 0.49, what is the probability that a given student owns neither a car nor a computer?

Example 8

Assume that following an injury you received from playing your favorite sport, you obtain and read information on new pain medications. In that information you read of a study that was conducted to test the side effects of two new pain medications. Use the following table to answer the questions and decide which, if any, of the two new pain medications you will use.

	Number of side effects in 12-week clinical trial		
Side effect	**Placebo** $n = 192$	**Drug A** $n = 186$	**Drug B** $n = 188$
Upper respiratory congestion	10	32	19
Sinus headache	11	25	32
Stomach ache	2	46	12
Neurological headache	34	55	72
Cough	22	18	31
Lower respiratory congestion	2	5	1

1. How many subjects were in the study?

2. How long was the study?

3. What were the independent and dependent variables under study?

4. What type of variables are they (qualitative or quantitative)?

5. What is the probability that a randomly selected person was receiving a placebo?

6. What is the probability that a person was receiving a placebo or drug A? Are these mutually exclusive events? What is the complement to this event?

7. What is the probability that a randomly selected person was receiving a placebo or experienced a neurological headache?

Homework: pg. 193 #2, 4, 5, 6, 8, 10, 11, 13, 15, 21, 22

Multiplication Rules and Conditional Probability

Multiplication rules can be used to find the probability of two or more events that occur in sequence.

Independent Events: two events A and B are independent if the fact that A occurs does not affect the probability of B occurring.

Examples of independent events

a. Buying a car and owning a house
b. Rolling a die and getting a two and rolling a second die and getting a three
c. Rolling a die and then drawing one card from a deck

Multiplication Rule 1: When two evens are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) * P(B)$$

Example 1:

A coin is flipped and a die is rolled. Find the probability of getting a tail on the coin and a 1 on the die.

Example 2:

A card is drawn from a deck and replaced; then a second card is drawn. Find the probability of getting a queen and then a king.

Example 3:

An urn contains 3 red balls, 5 blue balls, and 4 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted.

a. Find the probability of selecting two blue balls.

b. Find the probability of selecting a red ball and then a white ball.

Example 4:

If 58% of high school students said that they exercise regularly, find the probability that 3 randomly selected high school students will say that they exercise regularly.

Dependent Events: when the outcome or occurrence of the first event affects the outcome or occurrence of the second even in such a way that the probability is changed, the events are said to be dependent.

Examples of dependent events

a. Drawing a card from a deck, not replacing it, and then drawing a second card
b. Having high grades and getting a scholarship
c. Parking in a no parking zone and getting a ticket

Conditional Probability: probability that event B occurs after event A has already occurred – denoted P(B|A).

Multiplication Rule 2: When two events are dependent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) * P(B|A)$$

Example 5
A person owns a collection of 30 CDs, of which 5 are country music. If 2 CDs are selected at random, find the probability that both are country music.

Example 6
Three cards are drawn from an ordinary deck and not replaced. Find the probability of getting 3 hearts.

Example 7
A flashlight has 8 batteries, 3 of which are defective. If 2 are selected at random without replacement, find the probability that both are defective.

Example 8
In a scientific study there are 14 guinea pigs, 11 of which are pregnant. If 3 are selected at random without replacement, find the probability that none are pregnant.

<u>Formula for Conditional Probability</u>: The probability that the second event B occurs given that the first event A has occurred can be found by dividing the probability that both events occurred by the probability that the first event has occurred:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 9

The probability that Sam parks in a no-parking zone *and* gets a parking ticket is 0.07, and the probability that Sam cannot find a legal parking space and has to park in the no-parking zone is 0.28. On Wednesday, Sam arrives at school and has to park in a no-parking zone. Find the probability that he will get a parking ticket.

Example 10

A recent survey asked 131 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	**Yes**	**No**	**Total**
Male	**34**	**22**	**56**
Female	**9**	**66**	**75**
Total	**43**	**88**	**131**

a. Find the probability that the respondent answered yes, given that the respondent was a female.

b. Find the probability that the respondent was a male, given that the respondent answered no.

Example 11
87 students in a school cafeteria were asked if they favored a ban on smoking in the cafeteria. The results of the survey are shown in the table.

Class	**Favor**	**Oppose**	**No opinion**
Freshman	**17**	**22**	**7**
Sophomore	**29**	**8**	**4**

If a student is selected at random, find the following probability. Given that the student is a freshman, he or she opposes the ban.

<u>Using the complement</u>

Example 12
A coin is tossed 7 times. Find the probability of getting at least one head.

Example 13
It was reported that 4% of the ties sold are bow ties. If 4 customers who purchased a tie are randomly selected, find the probability that at least one purchased a bow tie.

Homework: pg. 209 #1, 2, 3, 7, 8, 10, 16, 17, 18, 19, 28, 29, 34, 40, 45

Counting Rules

Fundamental Counting Rule: In a sequence of *n* events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total number of possibilities of the sequence will be:

$$k_1 * k_2 * k_3 * * * k_n$$

Example 14
A paint manufacturer wishes to manufacture several different paints. The categories include

Color	**Red, blue, white, black, green, brown, yellow**
Type	**Latex, oil**
Texture	**Flat, semigloss, high gloss**
Use	**Outdoor, indoor, wood surface**

How many different kinds of paint can be made if a person can select one color, one type, one texture, and one use?

Example 15

There are 10 different statistics books, 6 different geometry books, and 4 different trigonometry books. A student must select one book of each type. How many different ways can this be done?

Example 16
The digits 0, 1, 2, 3, and 4 are to be used in a four-digit ID card. How many different cards are possible if repetitions are permitted?

What if repetitions are not allowed?

Factorial Formulas: For any counting n

$n! = n(n-1)(n-2)***1$

$0! = 1$

Example 17

Suppose a business owner has a choice of five different locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the five locations?

Permutation: an arrangement of n objects in a **specific order**

Permutation Rule: The arrangement of *n* objects in a **specific order** using *r* objects at a time is called a *permutation of n objects taking r objects at a time*. It is written as $_nP_r$ and the formula is

$$_nP_r = \frac{n!}{(n-r)!}$$

Example 18

An inspector must select 7 tests to perform in a certain order on a manufactured part. He has a choice of 10 tests. How many ways can he perform 7 different tests?

Example 19

A television news director wishes to use three news stories on an evening show. One story will be the lead story, one will be the second story, and the last will be a closing story. If the director has a total of nine stories to choose from, how many possible ways can the program be set up?

Example 20

How many different ways can a chairperson, a secretary, a treasurer, and an assistant chairperson be selected for a research project if there are nine scientists available?

Combination: a selection of distinct objects **without regard to order**

Combination Rule: the number of combinations of *r* objects selected from *n* objects is denoted by ${}_nC_r$ and is given by the formula

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

Example 21
A bicycle shop owner has 8 mountain bicycles in the showroom. The owner wishes to select 5 of them to display at a bicycle show. How many different ways can a group of 5 be selected?

Example 22
How many ways are there to select 3 bracelets from a box of 15 bracelets, disregarding the order of selection?

Example 23
In a club there are 8 women and 6 men. A committee of 4 women and 2 men is to be chosen. How many different possibilities are there?

Example 24
How many different tests can be made from a test bank of 16 questions if the test consists of 11 questions?

pg. 220 #1, 2, 6, 7, 8, 11, 13, 15, 19, 26, 27, 28, 34, 39, 41, 43, 48

Probability and Counting Rules

Example 25
Find the probability of getting 4 jacks when 5 cards are drawn from an ordinary deck of cards.

Example 26
A box contains 23 transistors, 5 of which are defective. If 4 are sold at random, find the probability that none is defective.

Example 28
There are 7 married couples in a tennis club. If 1 man and 1 woman are selected at random to plan the summer tournament, find the probability that they are married to each other.

Example 29
A parent-teacher committee consisting of 3 people is to be formed from 16 parents and 6 teachers. Find the probability that the committee will consist of all teachers.

Example 27
A combination lock consists of the 26 letters of the alphabet. If a 3−letter combination is needed, find the probability that the combination will consist of the letters MCH in that order. The same letter can be used more than once.

Homework: pg. 226 #1, 2, 3, 4, 8, 15