

Rational Expressions and Functions

A rational expression is the quotient of two polynomials.

Examples of Rational Expressions:

$$\frac{x^2 + 15x + 44}{x^2 - 16}, \quad \frac{3x^2 - 3x - 18}{x - 1}$$

Examples:

1. Find the value(s) for which $\frac{3}{2x-7}$ is undefined.

2. Find the value(s) for which $\frac{6}{x^2 - 5x + 6}$ is undefined.

3. Simplify: $\frac{15x^4}{21x^3}$.

4. Simplify: $\frac{(x+9)(x-2)(x-8)}{(x-2)(x-8)(x+3)}$

5. Simplify: $\frac{x^2 + 10x + 24}{x^2 - 2x - 48}$.

6. Simplify: $\frac{x^2 + 8x - 9}{1 - x^2}$.

7. Find the domain of $f(x) = \frac{x^2 + 11x + 24}{x^2 - 4x - 45}$

Homework: pg. 429 #15,17,19,21,23,25,27,29,31,33,35,37,45,47

Multiplication and Division of Rational Expressions

Examples:

Multiply.

1. $\frac{x^2 - 3x - 18}{x^2 - 9x + 20} \cdot \frac{x^2 - 3x - 10}{x^2 + 13x + 30}$

2. $\frac{36 - x^2}{x^2 + 9x + 14} \cdot \frac{x^2 - 7x - 18}{x^2 - 15x + 54}$

3. For $f(x) = \frac{x+4}{49-x^2}$ and $g(x) = \frac{x^2-10x+21}{x^2+x-12}$, find $f(x) \cdot g(x)$.

Examples:

Divide.

1. $\frac{x^2 - 4}{x^2 - 5x + 4} \div \frac{x^2 - 6x - 16}{x^2 + 3x - 28}$

2. $\frac{x^2 - 13x + 36}{x^2 - x - 12} \div \frac{9 - x}{x^2 + 8x + 15}$

3. For $f(x) = x^2 - 8x + 12$ and $g(x) = \frac{x^2 - 12x + 36}{x^2 - 11x + 10}$, find $g(x) \div f(x)$.

Homework: pg. 438 #5,7,9,11,13,15,17,19,23,25,27,29,31,33,35,37,39,41,43,45,51,53

Adding and Subtracting Rational Expressions with the Same Denominator

Examples:

Add.

1. $\frac{7}{x+3} + \frac{3}{x+3}$

2. $\frac{2x+54}{x^2+4x-32} + \frac{3x-14}{x^2+4x-32}$

3. $\frac{x^2+6x+4}{x^2+4x-21} + \frac{5x+24}{x^2+4x-21}$

Subtract.

1. $\frac{25}{6x+36} - \frac{17}{6x+36}$

2. $\frac{3x-7}{x-9} - \frac{20}{x-9}$

$$3. \frac{3x^2 + 6x - 37}{64 - x^2} - \frac{2x^2 + 17x - 61}{64 - x^2}$$

Add or Subtract.

$$1. \frac{x}{5x - 20} + \frac{4}{20 - 5x}$$

$$2. \frac{x^2 - 2x - 11}{x^2 - 16} - \frac{7x - 25}{16 - x^2}$$

Homework: pg. 446 #11,,13,15,17,21,27,29,31,33,35,37,41,43,51,53,55,57

Adding and Subtracting Rational Expressions with Unlike Denominators

Examples:

Find the Least Common Denominator (LCD) and add or subtract.

1. $\frac{5}{x^2 - 6x - 16} + \frac{1}{x^2 + 6x + 8}$

$$2. \frac{x-2}{x^2+x-56} + \frac{4}{x^2+10x+16}$$

$$3. \frac{x}{x^2 + 8x + 15} - \frac{5}{x^2 + 12x + 35}$$

$$4. \frac{x+5}{x^2+8x+12} + \frac{x+9}{x^2-36}$$

Homework: pg. 454 #21,23,25,27,29,31,33,35

Complex Fractions

Examples:
Simplify.

1.
$$\frac{1 - \frac{9}{x^2}}{1 + \frac{3}{x}}$$

$$2. \frac{x + \frac{3}{7}}{14 + \frac{6}{x}}$$

$$3. \frac{\frac{1}{64} - \frac{1}{x^2}}{\frac{1}{8} - \frac{1}{x}}$$

$$4. \frac{\frac{6}{x-4} + \frac{5}{x+7}}{\frac{x+2}{x-4}}$$

$$5. \frac{\frac{x^2 - 2x - 48}{x^2 + 7x - 30}}{\frac{x^2 + 7x + 6}{x^2 - 5x + 6}}$$

Homework: pg. 460 #11,13,15,17,23,25,27

Rational Equations

Examples:
Solve.

1. $\frac{6}{x} - \frac{1}{8} = \frac{7}{40}$

2. $1 = \frac{5}{x} + \frac{24}{x^2}$

$$3. \frac{7}{x-4} + 3 = \frac{2x-1}{x-4}$$

$$4. \frac{x+2}{x^2-3x-54} = \frac{2}{x^2-12x+27}$$

$$5. \frac{x}{x+5} + \frac{3}{x-7} = \frac{36}{x^2 - 2x - 35}$$

$$6. \frac{x+4}{x^2 + 5x - 14} + \frac{2}{x^2 + 3x - 10} = \frac{38}{x^2 + 12x + 35}$$

Homework: pg. 468 #15,17,19,21,23,27,31,35,37,39,41,43,45

1. The sum of the reciprocal of a number and $\frac{3}{8}$ is $\frac{19}{40}$. Find the number.

2. The sum of the reciprocal of a number and $\frac{4}{7}$ is $\frac{43}{63}$. Find the number.

3. One positive number is 9 less than another positive number. If two times the reciprocal of the smaller number is added to four times the reciprocal of the larger number, the sum is equal to 1. Find the two numbers.

4. One positive number is four larger than another positive number. If the reciprocal of the smaller number is added to six times the reciprocal of the larger number, the sum is equal to 1. Find the two numbers.

5. Nick drove 24 miles, one way, to deliver a package. On the way home he drove 20 miles per hour faster than he did on his way to deliver the package. If the total driving time was 1 hour for the entire trip, find Nick's driving speed on the way home.

6. Ellen ran 7 miles this morning. After running the first 3 miles, she increased her speed by 2 miles per hour. If it took her exactly 1 hour to finish her run, find the speed at which she was running for the last 4 miles.

Homework: pg. 481 #7,13,15,33