

Introduction to Factoring; Greatest Common Factor; Factoring by Grouping

Greatest Common Factor (GCF): the largest whole number that divides evenly into the numbers

Examples:

1. Find the GCF of 12 and 20.

2. Find the GCF of 48, 120, and 156.

3. Find the GCF of $12x^5$ and $28x^3$.

4. Find the GCF of $x^3y^4z^9$, $x^6y^2z^{10}$, and x^7z^4 .

Factoring out the GCF:

1. Factor $6x^4 - 42x^3 + 90x^2$

2. Factor $12x^3 + 6x - 3$

3. Factor $5x(x - 9) + 14(x - 9)$

4. Factor $7x(x - 8) - 6(x - 8)$

Factoring by Grouping – 4 Terms

1. $x^3 + 4x^2 + 7x + 28$

2. $2x^2 - 10x - 9x + 45$

3. $4x^3 - 36x^2 + x - 9$

4. $3x^2 + 24x - 12x - 96$

Homework: pg. 362 #5,9,13,17,19,21,25,27,31,37,39,41,43,45,51,53,59

Factoring Trinomials of the Form $x^2 + bx + c$ when the coefficient of x^2 is 1.

Examples: Factor

1. $x^2 + 7x + 10$

2. $x^2 - 11x + 30$

3. $x^2 + 9x - 36$

4. $5x^2 + 40x + 60$ (look for GCF)

5. $x^2 - x - 42$

6. $x^2 + 5x - 6$

7. $x^2 + 5x + 6$

8. $-x^2 + 14x - 48$ (look for GCF)

9. $x^2 + 10x + 25$

10. $x^2 + 9x + 12$

11. $x^2 - 4xy - 32y^2$

Homework: pg. 369 #5-53 every other odd (5,9,13,...)

Factoring Trinomials of the Form $ax^2 + bx + c$ ($a \neq 1$).

Examples: Factor

1. $2x^2 + 7x + 6$

2. $3x^2 + 13x + 12$

3. $3x^2 + 11x + 6$

4. $21x^2 - 18x - 3$

5. $4x^2 - 13x + 10$

6. $4x^2 - 25x + 36$

7. $16x^2 - 38x + 21$

Homework: pg. 377 #3-29 (odd), 31-65 (every other odd)

Factoring Special Binomials

Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$

Examples:

Factor.

1. $x^2 - 25$

2. $81x^2 - 100y^4$

3. $7y^2 - 252$ (look for GCF)

4. $64 - 9x^2$

5. $x^2 + 25$

Difference of Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Sum of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Examples:

Factor.

1. $x^3 - 125$

2. $x^{12} + 8$

3. $8z^3 + 27$

4. $64x^3 - 216$

Homework: pg. 384 #7-51 (every other odd), 53-87 (odd)

Factoring a Polynomial:

1. **Factor out any common terms.**
2. Determine the number of terms in the polynomial.
 - a. If there are only **two** terms, check to see if the binomial is one of the special binomials discussed in Section 6.4.
 - Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$
 - Sum of Squares: $a^2 + b^2$ is not factorable
 - Difference of Cubes: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
 - Sum of Cubes: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
 - b. If there are **three** terms, try to factor the trinomial using the techniques of Sections 6.2 and 6.3.
 - $x^2 + bx + c = (x+m)(x+n)$: find two integers m and n whose product is c and whose sum is b.
 - $ax^2 + bx + c$ ($a \neq 1$): factor using grouping, refer to object 1 in Section 6.3
 - c. If there are **four** terms, try factoring by grouping, discussed in Section 6.1.
3. After the polynomial has been factored, be sure that any factor with two or more terms does not have any common factors other than 1. If there are common factors, factor them out.
4. Check your factoring through multiplication.

Examples.

Factor.

1. $-6x^2 + 54x + 60$

2. $216x^{12} - 125$

3. $x^4 + 5x^3 - 8x - 40$

4. $x^6 + 64$

5. $4x^2 + 21x - 18$

Homework: pg. 390 # 7-61 (every other odd)

Solving Quadratic Equations by Factoring:

If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

Examples:

Solve.

1. $(x - 2)(x + 8) = 0$

2. $x(4x + 9) = 0$

3. $x^2 + 7x - 30 = 0$

4. $4x^2 + 60x + 224 = 0$

5. $6x^2 - 23x + 7 = 0$

6. $x^2 + 4x = 45$

7. $x(x-3) = 70$

8. Find a quadratic equation that has the solutions -6 and 8.

9. Find a quadratic equation that has the solutions -2 and -5.

Homework: pg. 398 #5-55 (odd),61,63,65

Quadratic Functions:

$$f(x) = ax^2 + bx + c.$$

Examples:

1. For $f(x) = x^2 - 9x + 12$, find $f(-6)$.

2. Let $f(x) = x^2 - 17x + 72$. Find all values for x so that $f(x) = 0$.

3. Let $f(x) = x^2 - 6x + 20$. Find all values for x so that $f(x) = 92$.

Homework: pg. 404 #3,5,7,9,21,23,25,27

3. One positive number is 9 more than a second number, and their product is 112. Find the two numbers.

4. One positive number is 3 more than twice a second number, and their product is 189. Find the two numbers.

5. The area of a rectangle is 105 square feet. If the length of the rectangle is 8 feet more than its width, find the dimensions of the rectangle.

6. A homeowner poured a rectangular concrete slab in her backyard to use as a barbecue area. The length is 3 feet more than its width. There is a 2-foot wide flower bed around the barbecue area. If the area covered by the barbecue area and the flower bed is 270 square feet, find the dimensions of the barbecue area.

7. A model rocket is launched from the ground with an initial velocity of 144 feet per second. The function $h(t) = -16t^2 + 144t$ give the height of the rocket, $h(t)$, in feet after t seconds.

a. How long will it take for the rocket to land on the ground?

b. What is the domain of the function $h(t)$?

Homework: pg. 413 #5,7,9,15,17,29,31,33,39,45,47