$Name$:____________________

242 - Differential Equations: Test 5
Henson - Summer 09

`DIRECTIONS`: **Show all work!** There is a total of 100 points.

1. a) Find a degree 4 Taylor polynomial that will APPROXIMATE the solution to the following differential equation around $x = 0$ with the given initial conditions:

$$y'' + y = 0 \text{ where } y(0) = 1 \text{ and } y'(0) = 0$$

   b) Looking at the D.E. you can see that $y(x) = cos(x)$ will be the exact solution that satisfies the D.E. and the initial conditions. Graph both the approximation from (a) and the exact solution $y(x) = cos(x)$ on your calculator and give a sketch. [Use the window xmin=-3, xmax=3, ymin = -1, ymax=1.5]

2. Find a power series solution to the following DE:

$$y' - xy = 0$$

3. Adjust the indices and powers of x to combine the following three series into a single series:

$$\sum_{n=0}^{\infty} a_n(n+1)x^n + \sum_{n=1}^{\infty} a_{n+2}(n-1)x^{n+1} + \sum_{n=2}^{\infty} a_n n(n+1)x^{n-2}$$

4. a) Take three derivatives of the following series, and then adjust the indices to start at zero again:

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   b) Identify f(x) using your answer from part (a).

5. Give a short 1-3 sentence answer to the following question: When solving for a power series solution to a homogeneous DE, why are we allowed to set each of the coefficients of x^k equal to zero?

6. Let $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$.

   a) Find the eigenvalues of A by hand.

   b) Find the eigenvectors of A corresponding to the eigenvalues found in (a) by hand.

7. $$\frac{dx}{dt} = 6x + 3y - 2z$$
   $$\frac{dy}{dt} = -4x - y + 2z$$
   $$\frac{dz}{dt} = 13x + 9y - 3z$$

   a) Write the above system in matrix notation $\vec{x}' = A \cdot \vec{x}$

   b) The eigenvalues of matrix A above are r=1, r=2, and r= -1. Find the three corresponding eigenvectors.

   c) Use the eigenvalues (I gave you) and eigenvectors (you found) to write the general solution to the system of DE's above.

8. Give a short 1-3 sentence answer to the following question: We often see the equation $A\vec{u} = r\vec{u}$ when solving systems of DE's. It seems that if you wanted to solve this equation, you could set this equation equal to zero. This would yield $A\vec{u} - r\vec{u} = 0$ which would imply $(A - r)\vec{u} = 0$. However, this will not work. What is wrong with expression $A - r$? How would you fix it?