Arc Length and Surfaces of Revolution

Goal: Find the length of a differentiable function from one point to another.

Arc Length

Let $f(x)$ be a differentiable function on the interval $[a, b]$. Then the arc length between a and b is:

$$S = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

(If $x = g(y)$, then $S = \int_c^d \sqrt{1 + [f'(y)]^2} dy$)

*Often these integrals are difficult to evaluate (may need the use of a calculator).

Ex: $y = x^2$ on $[2, 5]$

$f(x) = x^2$

$f'(x) = 2x$

$$S = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

$$S = \int_2^5 \sqrt{1 + 4x^2} dx$$

using calculator $\rightarrow S \approx 21.23$

Goal: Find the surface area of a solid of revolution.

Surface Area: Let $f(x)$ be a differentiable funtion on $[a, b]$. Then the surface area formed by revolving f around the x or y axis is:

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}dx$$

*NOTE: $r(x)$ is the distance to the axis of revolution

(alternate formula: $S = 2\pi \int_c^d r(y)\sqrt{1 + [g'(y)]^2}dy$)

Ex: $f(x) = x^2$ on $[0, 2]$

x-axis:

$$S = 2\pi \int_0^2 x^2\sqrt{1 + (2x)^2}dx$$

$$S = 2\pi \int_0^2 x^2\sqrt{1 + 4x^2}dx$$

using calculator $\rightarrow S \approx 8.47$