

Inverse Functions

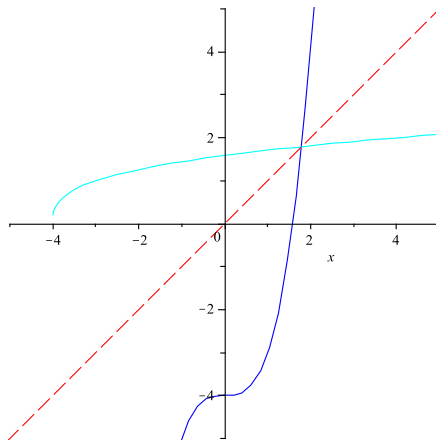
Let $f(x) = x + 2$ and $g(x) = x - 2 \rightarrow$ These are inverses

Let $f(x) = x^2$ and $g(x) = \sqrt{x} \rightarrow$ These are not inverses

Theorem: A function $f(x)$ has an inverse if f is a one-to-one function (passes the vertical and horizontal line tests)

Notation: $f^{-1}(x)$ means f -inverse

Graphing $f^{-1}(x)$: Reflect $f(x)$ across the line $y = x$



- Composition of $f(x)$ and $f^{-1}(x)$

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$$

Ex: $f(x) = \frac{1}{2}x - \frac{3}{2}$ and $g(x) = 2x + 3$

Finding $f^{-1}(x)$:

1. Let $y = f(x)$
2. Switch x and y
3. Solve for y
4. Replace y with $f(x)$

Ex:

$$f(x) = 3x - 5$$

$$y = 3x - 5$$

$$x = 3y - 5$$

$$y = \frac{1}{3}x + \frac{5}{3}$$

$$f(x) = \frac{1}{3}x + \frac{5}{3}$$