Natural Log Integration

Recall:

$$\frac{d}{dx}[ln|x|] = \frac{1}{x}$$

$$\frac{d}{dx}[ln|u|] = \frac{u'}{u}$$

This implies....

Log Rule:

$$\int \frac{1}{x}dx = ln|x| + C \text{ and } \int \frac{1}{u}du = ln|u| + C$$

*This rule uses u-substitution where the denominator is usually the u.

Ex:

$$\int \frac{10x}{5x^2 - 3}dx \leftarrow \text{Let } u = 5x^2 - 3 \text{ and } du = 10x$$

$$= \int \frac{du}{u} \text{ (or} \int \frac{1}{u}du) = ln|u| + C = ln|5x^2 - 3| + C$$

Tip: Many times the Log Rule is used when the degree of the numerator is one less than the denominator (except for trigonometric functions)

Ex:

$$\int \frac{1}{7x - 2}dx$$

Let $u = 7x - 2$ and $du = 7$

$$\int \frac{1}{7x-2}dx = \frac{1}{7}\int \frac{7}{7x-2}dx = \frac{1}{7}\int \frac{du}{u}$$

$$=\frac{1}{7}ln|u| + C = \frac{1}{7}ln|7x-2| + C$$

Note: Definite integrals work the same as in chapter 4.5 (u-substitution)