Natural Logarithm

Recall:

Exponentials: $y = b^x$

Logarithms: $y = log_b x$

(These are inverses)

In particular, $y = e^x$ and $y = ln(x)$

Properties of $ln(x)$:

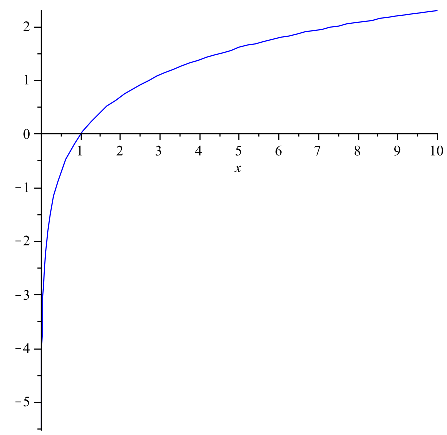
1. Domain is $(0, \infty)$, range is $(-\infty, \infty)$, and has x-intercept at (1,0)

2. Continuous, increasing, and one-to-one (passes verticle and horizontal line tests)

3. Concave down

- The natural logarithmic function is defined by:

$$ln(x) = \int_1^x \frac{1}{t}dt,\ x > 0$$



- 2nd Fundamental Theorem of Calculus says

if $F(x) = \int_a^x f(t)dt$, then $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$

so... $\frac{d}{dx}ln(x) = \frac{1}{x}$

Note that:

1. $f'(x) = \frac{1}{x} > 0$ since $x > 0$
2. $ln(x)$ is continuous since it is differentiable
3. $ln(x)$ is concave down since $f''(x) = \frac{-1}{x^2} < 0$ for $x > 0$

Derivative of $ln(x)$: Let u be a function of x

1. $\frac{d}{dx}ln(x) = \frac{1}{x}$

2. $\frac{d}{dx}ln(u) = \frac{1}{u} \cdot \frac{du}{dx} \leftarrow$ Chain rule