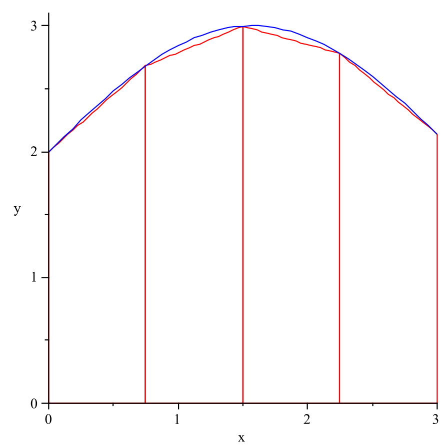
Numerical Integration

- Finding an antiderivative is not always possible, so we may need an approximation for a definite integral.
- One good approximation uses trapezoids

Trapezoidal Rule:

Steps to approximate area under $f(x)$ from $x = a$ to $x = b$



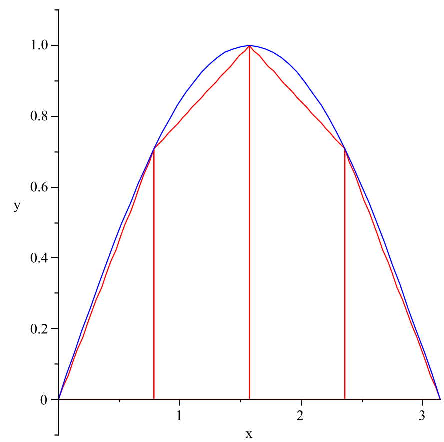
1. Partition $[a, b]$ as $a = x_0 < x_1 < ... < x_n = b$
2. Notice the area of the i^{th} trapezoid is (width) $\cdot$ (average height)= $\left(\frac{b-a}{n}\right) \cdot \left[\frac{f(x_{i-1}) + f(x_i)}{2}\right]$

Total Area= $\left(\frac{b-a}{n}\right) \cdot \left[\frac{f(x_0) + f(x_1)}{2} + ... + \frac{f(x_{n-1}) + f(x_n)}{2}\right]$

$$= \left(\frac{b-a}{2n}\right) \cdot [f(x_0) + 2f(x_1) + ... + 2f(x_{n-1}) + f(x_n)]$$

<u>Ex</u>: Approximate

$\int_0^{\pi} sin(x)dx$ using 4 trapezoids



$$\text{Area} \approx \left(\frac{b-a}{2n}\right) \cdot [f(x_0) + 2f(x_1) + ... + 2f(x_{n-1}) + f(x_n)]$$

$$\int_0^{\pi} sin(x)dx \approx \frac{\pi}{8}(sin(0) + 2sin(\frac{\pi}{4}) + 2sin(\frac{\pi}{2}) + 2sin(\frac{3\pi}{4}) + sin(\pi))$$

$$= \frac{\pi}{8}(0 + \sqrt{2} + 2 + \sqrt{2} + 0) = \frac{\pi(1+\sqrt{2})}{4} \approx 1.896$$