

Integration by Substitution

- Goal: to rewrite a complicated integrand in terms of u and du (simpler terms) in order to integrate.

Consider $\int f(g(x))g'(x)dx$. Let $u = g(x)$, so $du = g'(x)dx$

Therefore, $\int f(g(x))g'(x)dx = \int f(u)du$

Ex:

$$\int t\sqrt{t^2 + 2}dt$$

Let $u = t^2 + 2$, so $du = 2tdt...$ (you finish)

Change of Variables

1. Choose a u -substitution (usually the inside of a composite function)
2. Find $du = g'(x)$
3. Rewrite the integrand (in terms of only u)
4. Integrate
5. Write the final answer back in terms of x

Ex: $\int t\sqrt{t^2 + 2}dt$

$$u = t^2 + 2, du = 2t dt$$

$$\int t\sqrt{t^2 + 2} dt = \frac{1}{2} \int 2t\sqrt{t^2 + 2} dt$$

$$= \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} + C \right) = \frac{1}{3} (t^2 + 2)^{\frac{3}{2}} + C$$

Definite Integrals

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

**Be sure to rewrite the limits of integration