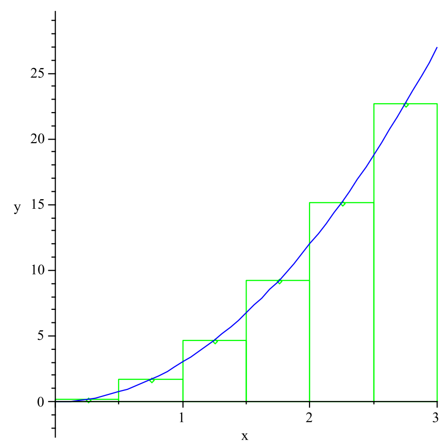
The Fundamental Theorem of Calculus

- Theorem used to compute the exact area under a curve

Recall:

$$\int_a^b f(x)dx \text{ (definite integral)} = \lim_{||\Delta|| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i \text{ (Riemann sum)}$$

Approximate $\int_0^3 3x^2 dx$ with an upper and lower sum using 6 rectangles.



$$\Delta x = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2}$$

Upper Sum:

$$S(n) = \sum_{i=1}^{6} f(M_i) \cdot \frac{1}{2}$$

$$= \frac{1}{2}(f(\frac{1}{2}) + f(1) + ... + f(3)) \approx \underline{?}$$

Lower sum:

$$s(n) = \sum_{i=1}^{6} f(m_i) \cdot \frac{1}{2}$$

$$= \frac{1}{2}(f(0) + f(\frac{1}{2}) + ... + f(\frac{5}{2})) \approx \underline{?}$$

<u>Theorem</u>: <u>The Fundamental Theorem of Calculus</u>

If f is continuous on $[a, b]$ and F is an antiderivative of f, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

*This gives a numerical answer (no $+C$)

<u>Notation</u>: $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

<u>Ex</u>:

$$\int_0^3 3x^2 dx = [x^3]_0^3 = 27 - 0 = \underline{27}$$

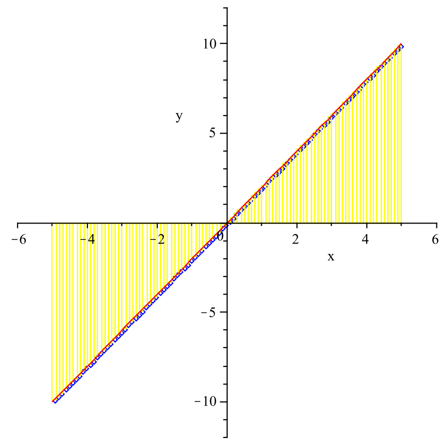
<u>Ex</u>:

$$\int_0^5 2xdx = [x^2]_0^5 = 25 - 0 = \underline{25}$$

$$\int_{-5}^5 2xdx = [x^2]_{-5}^5 = 25 - 25 = \underline{0}$$

$$\int_{-6}^4 2xdx = [x^2]_{-6}^4 = 16 - 36 = \underline{-20}$$

<u>Tips</u>:



- If $f(x)$ is symmetric, note that $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

- Average Value: $\dfrac{1}{b-a}\int_{a}^{b} f(x)dx$

2nd Fundamental Theorem of Calculus - if f is continuous, then $\dfrac{d}{dx}\int_{a}^{x} f(x)dx = f(x)$ for any a.