Riemann Sums/ Definite Integrals

Riemann Sum

Let f be defined on $[a, b]$, and let Δ ('delta') be a partition given by:

$a = x_0 < x_1 < ... < x_n = b$

where Δx_i is the i^{th} subinterval. If c_i is any point in the i^{th} subinterval, then

$$\sum_{i=1}^{n} f(c_i)\Delta x_i \ , \ x_{i-1} \leq c_i \leq x_i$$

is a Riemann sum for the partition Δ.

*We only use regular partitions (equal width subintervals)

Notation: $||\Delta||$ - "norm" of Δ (width of Δ's largest subinterval)

Definite Integral

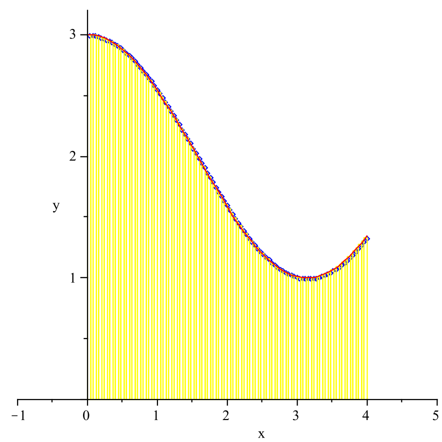
The given limit, $\lim_{||\Delta|| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$, is called the definite integral of f on $[a, b]$

CAUTION:

Indefinite integral - family of functions $(F(x) + \underline{C})$

Definite integral - gives a number (area under a curve)

Notice: $\int_a^b f(x)dx =$ Area under $f(x)$ from a to b

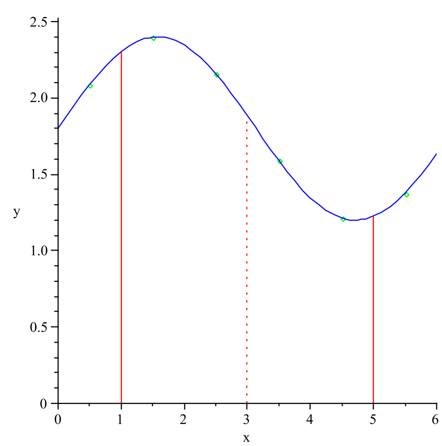


FACTS

1. $\int_a^a f(x)dx = 0$

2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

3. Let $a < c < b$. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$



4. $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

5. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$