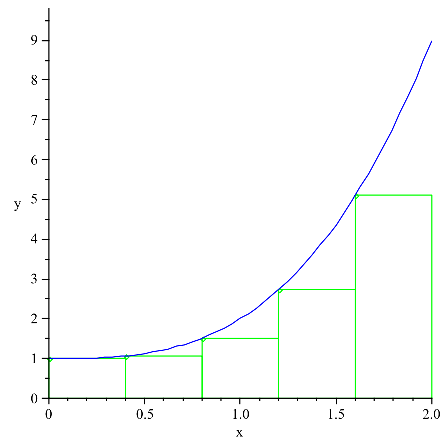
Area

- Main Idea: Approximate the area under a curve by taking the sum of separate rectangle areas.



Sigma notation (← shorthand notation for a sum)

$$\sum_{i=0}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

Ex:

$$\sum_{i=0}^{4} \frac{i}{i+1} = 0 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{163}{60} \approx 2.7$$

Summation Formulas

Add the areas of consecutive rectangles to approximate the exact area under the curve (use the lower sum).

Ex: $f(x) = x^3 + 1$ on interval $[0, 2]$ using 5 rectangles

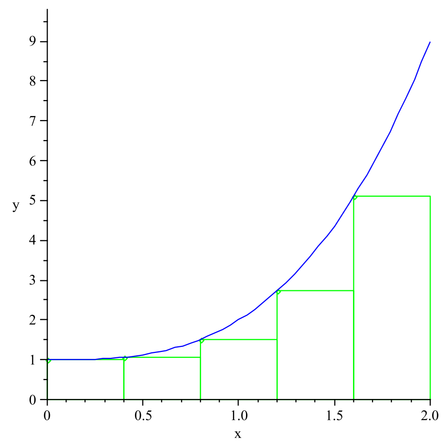
$\Delta x = \frac{b-a}{n} = \frac{2-0}{5} = \frac{2}{5}$ (width of each rectangle)

$s = \sum width \cdot height$

$s = \frac{2}{5} \cdot f(0) + \frac{2}{5} \cdot f(\frac{2}{5}) + \frac{2}{5} \cdot f(\frac{4}{5}) + \frac{2}{5} \cdot f(\frac{6}{5}) + \frac{2}{5} \cdot f(\frac{8}{5})$

$= \left(\frac{2}{5}\right)(1 + 1.064 + 1.512 + 2.728 + 5.096) = 4.56$

Actual area $\rightarrow$ 6



(Intervals: $\left[0, \frac{2}{5}\right], \left[\frac{2}{5}, \frac{4}{5}\right], ..., \left[\frac{8}{5}, \frac{10}{5}\right]$)

$\sum_{i=1}^{5} f\left(\frac{2i}{5}\right) \cdot \frac{2}{5}$ or $\sum_{i=1}^{5} f\left(\frac{2(i-1)}{5}\right) \cdot \frac{2}{5}$

Definition:

$f(m_i)$ - minimum value of f in the i^{th}h subinterval

$f(M_i)$ - maximum value of f in the i^{th} subinterval

$\Delta x = \frac{b-a}{n} \leftarrow$ Width of rectangles (constant!)

Lower sum $= s(n) = \sum_{1}^{n} f(m_i) \cdot \Delta x$

Upper sum $= S(n) = \sum_{1}^{n} f(M_i) \cdot \Delta x$

Notice: $s(n) \leq$ Actual Area $\leq S(n)$