Differentials

- NOTE: A tangent line can locally approximate a function $f(x)$.

Consider $f'(x)$ at $x = c$

Then, $f'(c) = \dfrac{y - f(c)}{x - c}$ ← tangent line at $x = c$

$y - f(c) = f'(c)(x - c)$

$y = f(c) + f'(c)(x - c)$ ← tangent line approximation to $f(x)$

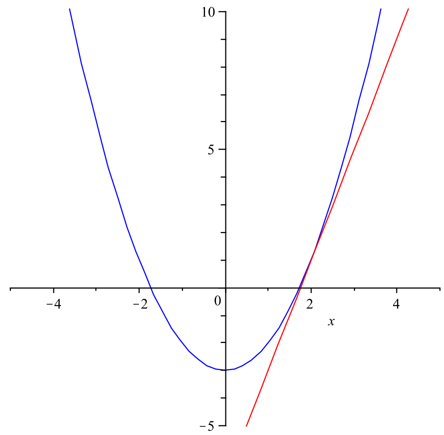
Compute the tangent line approximation for $f(x) = x^2 - 3$ at $(2, 1)$

$y = f(2) + f'(2)(x - 2)$

$y = 1 + 4(x - 2)$

$y = 4x - 7$

x	1.5	1.9	2	2.1	2.5
y=4x-7	?	?	1	?	?
$y = x^2 - 3$	?	?	1	?	?



- Differentials

Let $y = f(x)$. Then $\dfrac{dy}{dx} = f'(x)$

$dy = f'(x)dx \leftarrow$ the <u>differential</u> of y

*NOTE: $\Delta y \approx dy$ <u>and</u> $\Delta y \approx f'(x)dx$

<u>Approximating Function Values</u>

Notice:

$f(x + \Delta x) = f(x) + \Delta y$ (from $\Delta y = f(x + \Delta x) - f(x)$)

$\approx f(x) + dy$

$= f(x) + f'(x)dx$