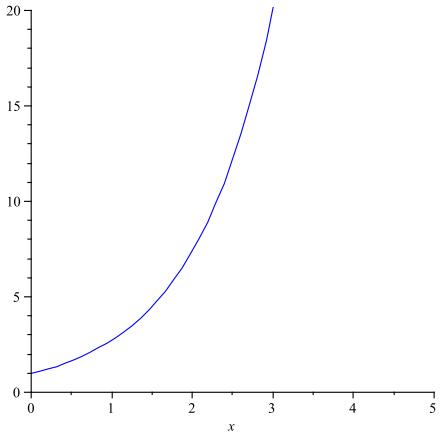
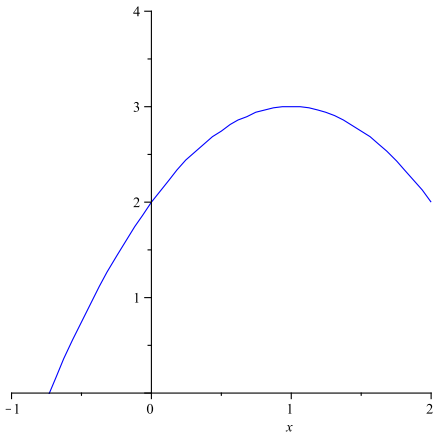
Concavity and 2nd Derivative Test

- Concavity

1. f is concave up when f' is increasing. *CAUTION: Notice I said f' is increasing and not f.



2. f is concave down when f' is decreasing



Test for Concavity

A function is concave up if $f''(x) > 0$.

A function is concave down if $f''(x) < 0$.

Ex: $f(x) = x^3 - 3x^2$

$f(x) = x^3 - 3x^2$

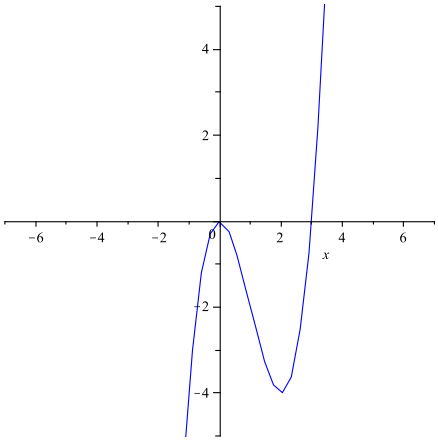
$f'(x) = 3x^2 - 6x = x(3x - 6)$

$x = 0$, $x = 2 \leftarrow$ critical values

$f''(x) = 6x - 6$

$f''(0) = -6$ (concave down at $x = 0$)

$f''(2) = 6$ (concave up at $x = 2$)



**Notice: $f(x)$ changes constantly only where $f''(x) = 0$ (or DNE)

- Inflection Points

Definition: $f(x)$ has a point of inflection at $(c, f(c))$ if f changes concavity at $x = c$.

Theorem: If $(c, f(c))$ is a point of inflection then $f''(c) = 0$ or DNE.

**Use possible points of inflection and a number line to determine concavity

- Second Derivative Test ($\leftarrow$ finds relative extrema)

Let f be a function such that $f'(c) = 0$. Then,

1. If $f''(c) > 0$, f has a relative min at $(c, f(c))$
2. If $f''(c) < 0$, f has a relative max at $(c, f(c))$
3. If $f''(c) = 0$, test does not work

Ex: $f(x) = x^3 - 12x + 9$

$f'(x) = 3x^2 - 12$

$f'(x) = 0 = 3x^2 - 12$

$x = 2$, $x = -2 \leftarrow$ critical values

$f''(x) = 6x$

$f''(2) = 12$ (relative min at $(2, -7)$)

$f''(-2) = -12$ (relative max at $(-2, 25)$)