Increasing and Decreasing Functions

- 1st Derivative Test

We are often concerned with where a function is increasing or decreasing. Specifically we look for the intervals on the x-axis where a function increases or decreases. Here we discuss how to find these intervals.

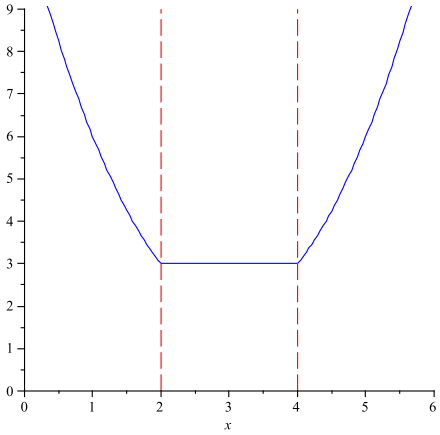
Definition:

f is increasing on an interval if for any x_1 and x_2, $x_1 < x_2$ implies that $f(x_1) < f(x_2)$.

f is decreasing on an interval if for any x_1 and x_2, $x_1 < x_2$ implies that $f(x_1) > f(x_2)$.

Notice:

1. f is increasing whenever $f'(x) > 0$
2. f is decreasing whenever $f'(x) < 0$
3. f is constant if $f'(x) = 0$



Ex: $f(x) = x^2$ so $f'(x) = 2x > 0$

So, in this case f is increasing when x is positive and decreasing when x is negative.

** While the idea of setting a derivative to be greater than zero technically always works, it is only practical to do this when f' is a very simple function. There is a better way to tell when a function is *increasing* or *decreasing*.

Notice: f changes direction (increasing/decreasing) only at critical values.

Ex:

$f(x) = x^3 - 3x^2$

Solve:

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = x(3x - 6)$

Therefore, Critical values are at $x = 0$ and $x = 2$.

- Finding Increasing/Decreasing Intervals

1. Locate all critical values ($f' = 0$ or DNE)

2. Plot critical values on a number line

3. Use test points to determine sign of f'

4. Determine increasing/decreasing intervals

Ex:

$$f(x) = \frac{x^5 - 5x}{5}$$

$$f'(x) = \frac{5(5x^4 - 5) - 0}{(5)^2}$$

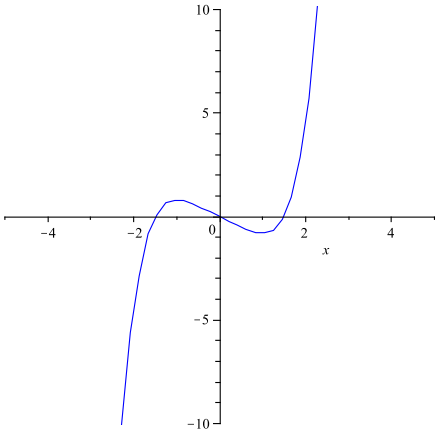
$$f'(x) = \frac{25x^4 - 25}{25}$$

$$f'(x) = x^4 - 1$$

$$f'(x) = 0 = x^4 - 1$$

$x = \pm 1 \leftarrow$ Critical values

$f(x)$ is increasing from $[-\infty, 1]$ and from $[1, \infty]$. $f(x)$ is decreasing from $[-1, 1]$.



Finding Extrema (Relative)

Theorem – 1st Derivative Test

Let f be continuous on (a, b). Then if:

1. $f'(x)$ changes from positive to negative at $x = c$, then f has a relative max at $(c, f(c))$.

2. $f'(x)$ changes from negative to positive at $x = c$, then f has a relative min at $(c, f(c))$.

   If $f'(x)$ has the same sign on both sides of c, then $(c, f(c))$ is not a max or min.

Ex:

$$f(x) = \frac{x^2 + 4}{x}$$