# Rolle's Theorem/Mean Value Theorem

*We would like to find points whose derivative matches the average slope of a function over an interval. The Mean Value Theorem allows us to do that, and Rolle's Theorem is a specific case of the Mean Value Theorem.*

Let's start with an exercise to illustrate Rolle's Theorem: Plot $(0, 2)$ and $(5, 2)$ – Draw any function you like between the two points.

Is there any x value, c, such that $f'(c) = 0$?

- Rolle's Theorem: Let $f$ be continuous on $[a, b]$ and differentiable on $(a, b)$.

  If $f(a) = f(b)$, then there is at least one $c$ in $(a, b)$ so that $f'(c) = 0$ in $(a, b)$.

  Ex: $f(x) = x^2 - 3x$ over $[0, 3]$

  $f(x) = \dfrac{x^2 - 1}{x}$ over $[-1, 1]$

  Doesn't work? (not continuous!)

- Mean Value Theorem(MVT): If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$ then there exists a number $c$ in $(a, b)$ such that,

  $$f'(c) = \frac{f(b) - f(a)}{b - a} \leftarrow \text{(average slope over the interval)}$$

- To find values that satisfy the MVT:

1. Find average slope over $[a, b]$
2. Set $f'(x) = \underline{1}$
3. Solve.

<u>Ex</u>: Find $c$ that satisfies the MVT:

- $f(x) = \dfrac{x+1}{x}$ over $[\frac{1}{2}, 2]$
- $f(x) = x^2 + 1$ over $[-1, 2]$