Extrema On An Interval

Extrema – Let f be defined on an interval I containing $x = c$.

1. $f(c)$ is the minimum if $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the maximum if $f(c) \geq f(x)$ for all x in I.

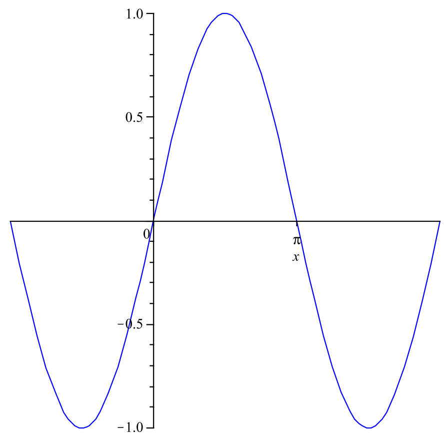
[Also called 'absolute min' and 'absolute max']

- Extreme Value Theorem – If f is continuous on a closed interval $[a, b]$, then f has both a max and min on the interval.

- Relative Extrema:

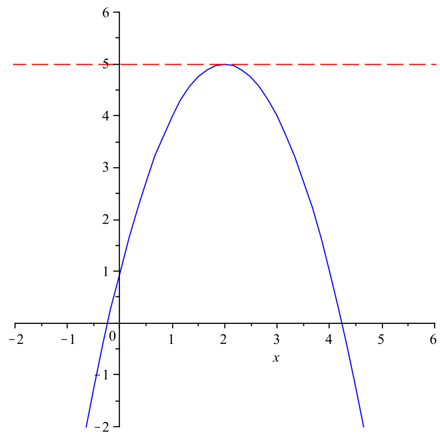
1. If there is any open interval where $f(c)$ is a max, then $f(c)$ is a relative max.
2. If there is any open interval where $f(c)$ is a min, then $f(c)$ is a relative min.

**Relative max/min is like the top/bottom of a hill.



Notice: Relative max/min only seem to occur when $f'(c) = 0$ or when $f'(c)$ does not exist.

Critical number – If $f'(c) = 0$ or $f'(c)$ does not exist, then c is a Critical number.



- Theorem: If f has a relative max/min at $x = c$, then c is a critical number.

- Finding absolute extrema on a closed interval $[a, b]$

1. List all critical numbers on $(a, b) \leftarrow$ open
2. Evaluate f at each critical number
3. Evaluate f at each endpoint of $[a, b]$
4. Highest number – max

   Lowest number – min

Ex:

- $f(x) = x^3 - \frac{3}{2}x^2$ on $[-5, 5]$

- $f(x) = \frac{2x}{x^2 + 1}$ on $[-3, 7]$