Related Rates

- Related Variables – Variables are related in an equation

[Recall: Implicit Differentiation]

$$\frac{d}{dx}[4x^2] = 8x$$

$$\frac{d}{dx}[y^3 + 2x] = 3y^2\frac{dy}{dx} + 2$$

- In many applications, variables change with respect to time (time is the independent variable, not x). $\leftarrow$ requires $\frac{d}{dt}$ not $\frac{d}{dx}$

Ex:

$$\frac{d}{dt}[4] = 0$$

$$\frac{d}{dt}[x^2] = 2x\frac{dx}{dt}$$

$$\frac{d}{dt}[5y + 3x^2] = 5\frac{dy}{dt} + 6x\frac{dx}{dt}$$

Ex: $y = 2x \rightarrow$ distance

$$\frac{dy}{dt} = 2\frac{dx}{dt}$$

Steps to solve:

1. Write equation relating all variables

2. Differentiate with respect to time t

3. Plug in all known values

Ex: Inflating a Balloon

A balloon (sphere) is being blown up with volume increasing at a rate of 4 inches^3/second. At what rate is the radius increasing when the diameter is 3 inches?

$V = \frac{4}{3}\pi r^3 \leftarrow$ Volume of a sphere

$\frac{dr}{dt} = ?$ $\quad \frac{dV}{dt} = 4$ $\quad r = 1.5$

1. Differentiate volume formula $\rightarrow$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

2. Solve for $\frac{dr}{dt} \rightarrow$ $\quad \frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$

3. Plug in values $\rightarrow$ $\quad \frac{dr}{dt} = \frac{4}{4\pi(1.5)^2} \approx 0.141\ in^3/sec$

- Formulas to remember:

1. Area (triangles, circles, rectangles)

2. Volume (spheres)

3. Pythagorean Theorem