Implicit Differentiation

Recall: To be considered a function, must pass the Verticle Line Test

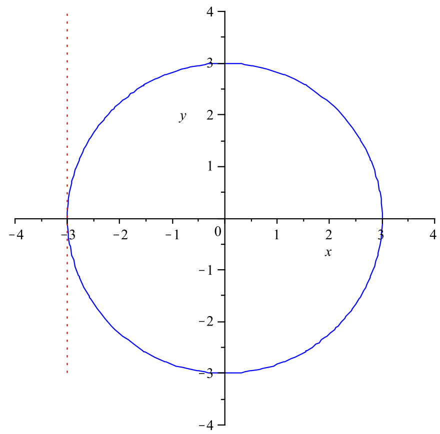
Ex: $y = x^2 - 1$ or $y = sin(x)$

↑ Notice: All functions are explicitly in terms of x.

So... how could we get the derivative of:

$$x^2 + y^2 = 9$$

since y is not solved explicitly in terms of x??



To differentiate equations that have terms containing y, we need something called implicit differentiation. When you differentiate a term with x, the rules stay the same. However, when you differentiate a term with y, follow the derivative with $\frac{dy}{dx}$. Consider some of the following trivial examples.

$$\frac{d}{dx}[5x^2] = 10x\frac{dx}{dx} = 10x$$

$$\frac{d}{dx}[x^3 - 5x] = 3x^2\frac{dx}{dx} - 5\frac{dx}{dx} = 3x^2 - 5$$

$$\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$$

$$\frac{d}{dx}[y = x^2 - 3x + 2] \to \frac{dy}{dx} = 2x - 3$$

$$\frac{d}{dx}[xy^3] = x(3y^2\frac{dy}{dx}) + y^3(1)$$

<u>Steps for Implicit Differentiation</u>

1. Differentiate both sides with respect to x
2. Collect all $\frac{dy}{dx}$'s on one side, everything else to the other side.
3. Common factor: $\frac{dy}{dx}$.
4. Solve for $\frac{dy}{dx}$.

<u>Ex</u>:

$$x^2y + y^2x = -2$$

$$(x^2\frac{dy}{dx} + 2xy) + (y^2 + 2y\frac{dy}{dx} \cdot x) = 0$$

$$x^2\frac{dy}{dx} + 2xy + y^2 + 2xy\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$