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# The Chain Rule

*The chain rule deals with differentiating composite functions. [Ex: $f(g(x))$]*

Theorem- If $y = f(u)$ is a differentiable function of $u$ and $u = g(x)$ is a differentiable function of $x$, then $y = f(g(x))$ is a differentiable function of $x$.

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Ex:

Differentiate $y = sin(2x)$.

$$\frac{dy}{dx} = cos(2x) \cdot (2) = 2cos(2x)$$

The General Power Rule (special case of chain rule)

Theorem- Let $y = [u(x)]^h$, where $u$ is a differentiable function of $x$ and $n$ is a rational number. Then,

$$\frac{d}{dx}[u^n] = n \cdot u^{n-1} \cdot u'$$

Ex:

Differentiate $y = (x^3 + 2x)^2$.

$$\frac{d}{dx} = 2(x^3 + 2x)^1 \cdot (3x^2 + 2)$$

Ex: Differentiate $y = (\frac{3x-1}{x^2+3})^2$

$$y' = 2(\frac{3x-1}{x^2+3})^1 \cdot \frac{d}{dx}[\frac{3x-1}{x^2+3}]$$

$$y' = [\frac{2(3x-1)}{x^2+3}] \cdot [\frac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)^2}]$$

Simplify:

$$y' = \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$$

Trigonometric Functions

$$\frac{d}{dx}[sinu] = (cosu) \cdot u'$$

$$\frac{d}{dx}[cosu] = -(sinu) \cdot u'$$

$$\frac{d}{dx}[tanu] = (sec^2u) \cdot u'$$

$$\frac{d}{dx}[cotu] = -(csc^2u) \cdot u'$$

$$\frac{d}{dx}[secu] = (secu \cdot tanu) \cdot u'$$

$$\frac{d}{dx}[cscu] = -(cscu \cdot cotu) \cdot u'$$