Product and Quotient Rule/ Higher Order Derivatives

To differentiate the product or quotient of two or more functions, you CANNOT simply differentiate all functions individually. You need the product rule or the quotient rule.

- Product Rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Ex:

1. $\frac{d}{dx}\left[(2x^2+1)(x-3)\right]$

   $= (4x)(x-3) + (2x^2+1)(1) = 4x^2 - 12x + 2x^2 + 1 = 6x^2 - 12x + 1$

2. $\frac{d}{dx}\left[x^2 \cdot sin(x)\right]$

   $= 2x \cdot sin(x) + x^2 \cdot cos(x) = 2xsin(x) + x^2cos(x)$

What about $\frac{d}{dx}[f(x) \cdot g(x) \cdot h(x)]$?

$= f(x) \cdot g(x) \cdot h'(x) + f(x) \cdot g'(x) \cdot h(x) + f'(x) \cdot g(x) \cdot h(x)$

- Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

[lo d hi less hi d lo..and down below]

Ex:

$$\frac{d}{dx}\left[\frac{2x-3}{5x+1}\right] = \frac{(5x+1)(2)-(2x-3)(5)}{(5x+1)^2}$$

$$= \frac{10x+2-10x+15}{25x^2+10x+1} = \frac{17}{25x^2+10x+1}$$

CAUTIONS:

1. Simplify compound fractions before using quotient rule.

2. When dividing by a constant, do not use the quotient rule.

3. It is sometimes easier to rewrite the equation using negative exponents.

Trigonometric Functions

Recall:

$$\frac{d}{dx}[sin(x)] = cos(x)$$

$$\frac{d}{dx}[cos(x)] = -sin(x)$$

$$\frac{d}{dx}[tan(x)] = sec^2(x)$$

$$\frac{d}{dx}[sec(x)] = sec(x)tan(x)$$

$$\frac{d}{dx}[cot(x)] = -csc^2(x)$$

$$\frac{d}{dx}[csc(x)] = -csc(x)cot(x)$$

Remember your basic Trig Identities:

$sin^2(x) + cos^2(x) = 1$

$2sin(x)cos(x) = sin(2x)$

$cos^2(x) - sin^2(x) = cos(2x)$

- Higher Order Derivatives ← (more than 1 derivative)

First Derivative: $f'(x)$, $\dfrac{dy}{dx}$

Second Derivative: $f''(x)$, $\dfrac{d^2y}{dx^2}$

Third Derivative: $f'''(x)$, $\dfrac{d^3y}{dx^3}$

n^{th} Derivative: $f^n(x)$, $\dfrac{d^ny}{dx^n}$

*Real life application Physics:

$s(t)$ - Position

$v(t) = s'(t)$ - Velocity

$a(t) = v'(t) = s''(t)$ - Acceleration