Basic Differentiation Rules

Instead of using the long limit definition for derivatives, there are a list of a dozen or so rules that provide shortcuts to taking derivatives.

- Theorem: The constant rule

  $$\frac{d}{dx}[c] = 0$$

  Ex: $\frac{d}{dx}[10] = 0$

  Guess the pattern:

  $f(x) = x^1 \to f'(x) = 1$

  $f(x) = x^2 \to f'(x) = 2x$

  $f(x) = x^3 \to f'(x) = 3x^2$

- Theorem: The power rule

  Let n be a rational number.

  $$\frac{d}{dx}[x^n] = nx^{n-1}$$

  Ex: $\frac{d}{dx}[x^6] = 6x^5$

  **CAUTION: You may need to rewrite $f(x)$ to have negative exponents

Ex: $\frac{d}{dx}[\frac{1}{x^5}] = \frac{d}{dx}[x^{-5}]$

$= [-5x^{-6}] = \frac{-5}{x^6}$

- Find the equation of a tangent line:

1. Take $f'(x)$
2. Compute slope at $x = a$
3. Use slope and $(a, f(a))$ to determine line

Ex: Find tangent line of $f(x) = x^2$ at $x = -2$

$(-2, f(-2)) = (-2, 4)$

$f(x) = x^2$

$f'(x) = 2x$

$f'(-2) = -4 \leftarrow$ (slope)

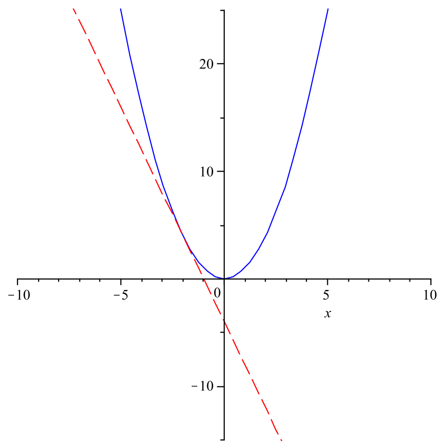
$(y - y_1) = m(x - x_1)$

$y - 4 = -4(x - (-2))$

$y = -4x - 4 \leftarrow$ Equation of tangent line

- Theorem: Constant multiple rule

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$$



- Theorem: Sum and Difference rules

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

[**This allows us to differentiate polynomials!]

Ex: $\frac{d}{dx}[x^3 + 2x] = 3x^2 + 2$

Some trigonometric derivatives:

$$\frac{d}{dx}[sin(x)] = cos(x)$$

$$\frac{d}{dx}[cos(x)] = -sin(x)$$

Physics

Let $s(t)$ represent an object's position at time t.

$s(t)$ – position

$v(t) = s'(t)$ – velocity

$a(t) = v'(t)$ – acceleration

Position of a free falling object is given by:

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

where, v_0 is initial velocity, s_0 is initial position, and g is earth's acceleration due to gravity (about $-32ft/sec^2$ or $-9.8m/sec^2$)

**CAUTION:

Instantaneous velocity at $x = c$ means find $f'(c)$.

Average velocity over an interval $[a, b]$ means the average slope : $\frac{rise}{run} = \frac{f(b) - f(a)}{b - a}$

Ex: Section 2.2, number 93