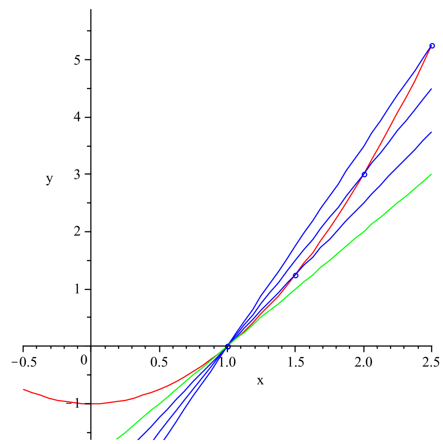
Tangent Line Problem

We would like to find the equation of a line that barely touches a function f(x) at a point x=c. This line is called the tangent line of f(x) at x=c.

- A secant line is a line between two points on a curve.



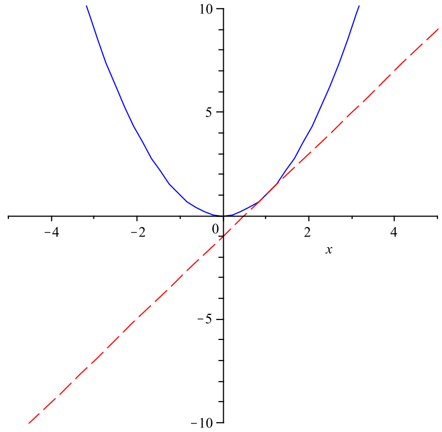
- We can approximate the slope of a tangent line to a curve using a secant line.

- Δx – change in $x \leftarrow$ also called h

  Δy – change in y

- Definition: The slope of a tangent line at a point x is given by $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ (Difference Quotient)

- For example, if $f(x) = mx + b$, then $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = m$



Ex: $f(x) = 2x - 3$ [You try...]

**Notice here the slope did not depend on x.

- Nonlinear functions – same definition applies

  Ex: $f(x) = x^2 + 3$ [You try...]

- Derivative–

  The derivative of f at x is:

  $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

  $[f'(x) \to f \textit{ prime of } x]$

  The process of finding the derivative is called differentiation

  Notation: $f'(x),\ \frac{dy}{dx},\ y',\ \frac{d}{dx}f(x),\ D_x[y]$

$\frac{dy}{dx}$ *is the derivative of* y *with respect to* x.

- Finding the slope of a function when $x = c$.

1. Take the derivative (find f')
2. Evaluate at $x = c$

**CAUTION: If $f(x)$ has a 'sharp turn' at $x = c$, then we say $f'(c)$ DNE (does not exist).

- Theorem: If f is differentiable at $x = c$, then f is continuous at c.

Therefore...

1. Differentiability implies continuity
2. Continuity does not imply differentiability