Continuity/One Sided Limits

- A function is continuous if it can be drawn without lifting the pencil.

Definition: Let c be in the open interval (a, b). Then, $f(x)$ is continuous at $x = c$ if:

1. $\lim_{x \to c} f(x)$ exists
2. $f(c)$ exists
3. $\lim x \to c f(x) = f(c)$

Ex: Prove $f(x) = x^2 + 3$ is continuous at $x = 1$.

Types of Discontinuities:

- Removable - redefining 1 point will make $f(x)$ continuous
- Nonremovable - redefining 1 point will not make $f(x)$ continuous

One Sided Limits (used in taking limits on closed intervals)

$\lim_{x \to c^-} f(x) \leftarrow$ limit as $x \to c$ from the left

$\lim_{x \to c^+} f(x) \leftarrow$ limit as $x \to c$ from the right

Theorem: $\lim_{x \to c} f(x)$ exists if and only if the limits from the left and the right agree.

Intermediate Value Theorem

If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there exists some c so that $f(c) = k$

Helpful in showing roots exist in functions

Ex: Root between $x = 1$ and $x = 2$

$f(x) = \frac{1}{16}x^4 - x^3 + 3$ $[1, 2]$