Evaluating Limits Analytically

- We can use substitution to evaluate limits for well behaved functions.
- In General: $\lim_{x \to c} f(x) = f(c)$

Ex: $\lim_{x \to 3} x^2 = 9$

Theorem: Let b and c be real numbers and let n be a positive integer.

1. $\lim_{x \to c} b = b$
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c} x^n = c^n$

Functions that allow substitution:

$\lim_{x \to c} f(x) = f(c)$

1. Polynomials - $p(x)$
2. Rational functions - $\dfrac{p(x)}{q(x)}$ if $q(c) \neq 0$
3. Trigonometric functions - $sin(x)$, $tan(x)$, ...

Properties of Limits

- Let $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$

1. $\lim_{x \to c} c \cdot f(x) = c \cdot L$
2. $\lim_{x \to c} [f(x) \pm g(x) = L \pm K$
3. $\lim_{x \to c} [f(x) \cdot g(x)] = L \cdot K$
4. $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, $K \neq 0$
5. $\lim_{x \to c} [f(x)]^n = L^n$

<u>FACT</u>: you are allowed to cancel any common factors <u>before</u> taking a limit.

<u>Ex</u>: $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} \dfrac{(x+3)(x-2)}{(x-2)} = \lim_{x \to x} x + 3 = 5$

<u>Squeeze Theorem</u>

Let $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c.

If $\lim_{x \to c} g(x) = L = \lim_{x \to c} h(x)$, then $\lim_{x \to c} f(x) = L$ also.

<u>Ex</u>:

$\lim_{x \to 0} f(x)$ where $4 - x^2 \leq f(x) \leq 4 + x^2$

$\lim_{x \to c} f(x) = 4$

<u>Special Limits</u>

1. $\lim_{x \to 0} \frac{sin(x)}{x} = 1$

2. $\lim_{x \to 0} \frac{1 - cos(x)}{x} = 0$