Finding Limits Graphically and Numerically

A limit asks what a function $f(x)$ approaches as x approaches a constant. This limit can be evaluated 3 ways: graphically, numerically, and analytically. The first two methods are discussed here.

Basic Notation: $\lim_{x \to c} f(x) = L$

[*The limit of f(x), as x goes to c, is equal to L.*]

Example: As $x \to 2$, the function $f(x) = 3x - 4$ approaches 2.

or $\lim_{x \to 2} f(x) = 3x - 4 = 2$

[*The limit as x approaches 2 of* $f(x) = 3x - 4$ *equals 2.*]

**NOTE: The limit as $x \to c$ has nothing to do with the function's value at c. We are only c

Example: (Numerically)

Question: As $x \to 0$, then $f(x) = \frac{\sin x}{x}$ approaches ??

*Notice we cannot plug in zero (division by zero), but again, this does not mean the limit does not exist. The limit is not concerned with what happens at zero, but what happens as x approaches zero.

Make a table:

x	-0.2	-0.1	-0.01	0.01	0.1	0.2
f(x)	?	?	?	?	?	?

Plug in values:

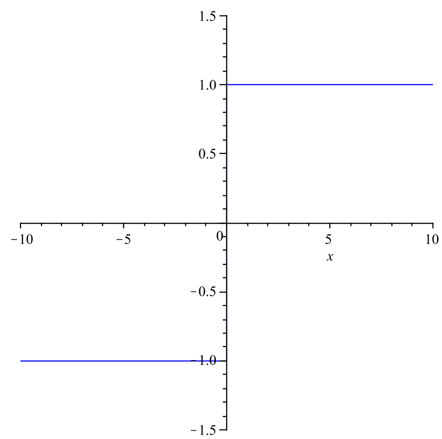
x	-0.2	-0.1	-0.01	0.01	0.1	0.2
f(x)	0.9933	0.9983	0.9999	.9999	.9983	.9933

So, As $x \to 0$ $f(x)$ approaches 1.

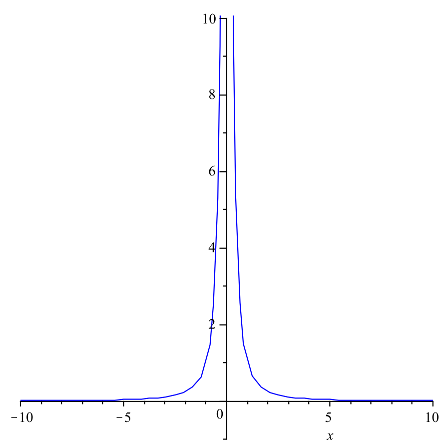
**NOTE: The limits from the left and right must be equal for a limit to exist.

Common Types of Nonexistant Limits

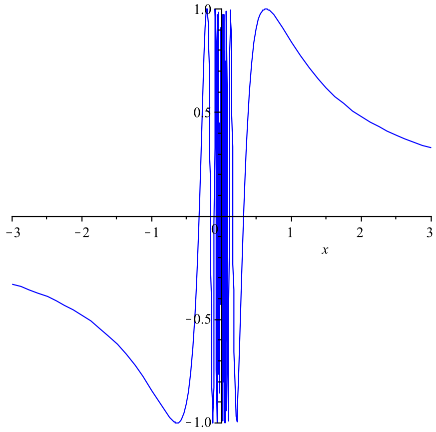
1. Limits that differ from Left and Right



2. $f(x)$ increases/decreases without bound as $x \to c$.



3. $f(x)$ oscillates between two fixed values as $x \to c$.



Formal Definition of a Limit

States that the $\lim_{x \to c} f(x) = L$ if and only if for every $\epsilon > 0$, there exists some $\delta > 0$ so that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

Notation: ϵ - epsilon , δ - delta

**RECALL: $|b - a|$ is the distance between b and a.